

gods themselves; and that, and only that, is what makes us human. Suitably spoke Socrates on this subject: “For the soul which has never seen the truth, can never pass into human form. For a human being must understand a general conception formed by collecting into a unity by means of reason, the many perceptions of the senses; and this is a recollection of those things which our soul once beheld, when it journeyed with God and, lifting its vision above the things which we now say exist, rose up into real being.”

BOY: So it wasn't really the case, as they say, that Prometheus ran out of gifts of abilities and talents, after bestowing them on all the other creatures, leaving humankind with nothing else but fire?

HERDSMAN: Now you are starting to perceive the true meaning of our origin. For it is true that we were not given any physical prowess, as that had already been

doled out to the animals. But Prometheus knew that none of these creatures had bodies that could house a spirit capable of governing the world around. So, crafty Prometheus had the foresight to know that the seed of heaven lay sleeping in the Earth. He scooped up some clay, moistened it with water from a river, and kneaded it this way and that, and shaped it into the image of the gods. In order that we might have life, the core of many animals was locked in our breasts; but our true nature came only after Athena marvelled at it all, and breathed into us the spirit, the divine breath, which made us completely alive. Even so, we aimlessly moved about. We saw, yet we did not see; we heard, yet we did not hear. We wandered as figures in a dream, until Prometheus stole for us the sacred prize of Olympus, fire. Then, when we sheltered ourselves from the elements with our gift of fire, and our eyes started to follow the smoke up into the night sky, did our souls float upward following after. For then Prometheus taught us about the rising and the setting of the stars, discovering for us the art of counting in true number, and communicated to us the music of poetry.

## The Legacy of Plato

A physical concept of magnitude was already fully developed by the circle associated with Plato, and expressed most explicitly in the *Meno*, *Thaetetus*, and *Timaeus* dialogues. Plato and his circle demonstrated this concept, pedagogically, through the paradoxes that arise when considering the uniqueness of the five regular solids, and the related problems of doubling a line, square, and cube. As Plato emphasized, each species of action generated a different species of magnitude. He denoted such species by the Greek word *dunamis*, the root of the English ‘dynamo,’ translated as ‘power.’ The meaning of the term *dunamis* is akin to Leibniz’s use of the German word *Kraft*.

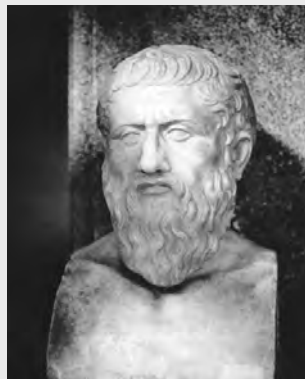
That is, a linear magnitude has the *power* to double a line, whereas only a magnitude of a different species has the *power* to double a square, and a still different species has the *power* to double a cube. In Bernhard Riemann’s terminology, these magnitudes are called, respectively, simply-extended, doubly-extended, and

triply-extended.

Plato’s circle emphasized that magnitudes of lesser extension lacked the potential to generate magnitudes of higher extension, creating, conceptually, a succession of higher *powers*.

Plato’s circle also emphasized, that this succession of magnitudes of higher powers, was generated by a succession of different types of action.

Specifically, a simply-extended magnitude was produced from *linear action*, doubly-extended magnitudes from *circular action*, and triply-extended magnitudes from *extended circular action*, such as the rotational actions that produce a cone, cylinder, or torus. Plato’s collaborator, Archytas, demonstrated that the magnitude with which a cube is doubled, is not generated by circular action, but by extended circular action, i.e., conic sections.



Plato (427-347 B.C.)

© IRNS/Philip Ulanowsky

—Bruce Director

BOY: But what about the barbarians, who never look up?  
What do they have to live for?

HERDSMAN: Do you know of the demi-god Asclepius?

BOY: Assuredly, he was raised by the centaur Chiron and became the greatest master of the art of medicine. Socrates spoke of his importance even in his dying words.

HERDSMAN: But did you know that he also goes by another name, a name from his native land?

BOY: What name is that?

HERDSMAN: He is the great Egyptian city-builder, physician, and the father of the Great Pyramid, Imhotep. You see my boy, while it is true that we are most favored and fortunate to be Greeks, do not think that all the barbarians have always been as they are now. We are not the first group of people to study the sphaerics of the heavens, and we will not be the last, either. For whenever men wish to advance the cause of humanity, they first need to attempt to contemplate the eternal. And that humbling act, as we were just humbled now, produces all the curiosity needed to investigate the true nature of all physical things. Indeed, astronomers know that there are Vedic peo-

ples who have stories of the heavens that are older than time itself, and our own ancestors from Egypt succeeded in unfathomable deeds without which we would not even exist. And, furthermore, that no civilization could ever have even survived without this kind of understanding.

BOY: That sounds beautiful, but I can not help but think that I can imagine one society that would not: What about a simple group of farmers, who would only have need to know how to plant a seed, and how to water it?

HERDSMAN: And also *where* to plant it, insofar as is best?

BOY: Certainly.

HERDSMAN: Then, I take it they would also need to know *when* to plant it, insofar as is best?

BOY: I suppose they would.

HERDSMAN: Then, suppose they needed to plant a certain seed in a certain place, right at the beginning of the season called spring, how would they know when to do the planting?

BOY: It wouldn't take an astronomer to count the phases of the moon. Just find the right time to plant, then wait 12 cycles of the moon, and you would be back at the beginning of spring.

HERDSMAN: Except that what actually determines our

## Archytas's Construction For Doubling the Cube

*Archytas developed a construction to find two geometric means between two magnitudes, AC and AB. Magnitude AC is drawn as the diameter of circle ABC; AB is a chord of the circle. Using this circle as the base, generate a cylinder. The circle is then rotated 90° about AC, so it is perpendicular to the plane of circle ABC; it is then rotated about point A, to form a torus with nil diameter. (The intersection of the torus and the cylinder produces a curve of double curvature.) Chord AB is extended until it intersects the perpendicular to AC at point D; this forms triangle ACD, which lies in plane of circle ABC, AB, and AC. Triangle ACD is then rotated around AC, producing a cone. The cone, torus, and cylinder all intersect at point P. Perpendicular PM is then dropped from P along the surface of the cylinder, until it intersects circle ABC at point M; this forms right triangle AMP. Through this construction, a series of similar right triangles (only partially shown) is generated, which produces the continued proportion,  $AB:AM::AM:AP::AP:AC$ . Thus, AM and AP are shown to be the two geometric means between magnitudes AC and AB.*

