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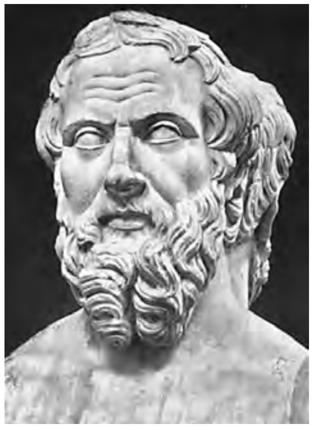
The First Measurement of the Universe

The spread of mythologies in the name of "history of science," began very early.

The Greek historian Herodotus reported that geometry was invented in Egypt, and transmitted from there to the Ionians. He also claimed, however, that geometry arose in connection with the practical problem of measuring and reconstructing the division-boundaries of agricultural fields after each periodic flooding of the Nile (geo-metry = earth-measurement). If Herodotus intended the term, "geometry," to signify some specialized knowledge relevant to surveying, there may be an element of truth to the latter assertion; but, if he meant the geometry of Thales, Anaximander, Pythagoras, and Plato, then the account is certainly wrong and highly misleading. This story of geometry's alleged practical origin (whether Herodotus is to blame for it or not), found its way into the subsequent histories of science, up to this day. It reminds us of the theory of the "opposable thumb" and other absurdities of Friedrich Engels' "dialectical materialism."

Contrary to this, the overwhelming evidence—including that contained in Plato's *Timaeus,* in the Vedic and other ancient calendars, as well as the implied navigational skills of the "Peoples of the Sea"—demonstrates that *all physical science originated in astronomy*. Astronomy, in turn, was cultivated in some form already tens, probably hundreds of thousands of years before the classically recorded Egyptian civilization, by maritime cultures spread across the globe. Geometry begins with nothing less, than man's attempt to measure the Universe as a whole.



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Herodotus (c. 484-430 B.C.)

This should indicate that the practice of basing school mathematics education on so-called "plane and solid geometry"—a practice that has dominated European education, despite the Renaissance, for over two millennia—is profoundly in error. Henceforth, the teaching of geometry should begin with the *failure* of plane and solid geometry to account for the most elementary features of visual astronomy. That failure has a precise, knowable structure; to characterize that singularity, is to carry out the first scientific measurement of the Universe.

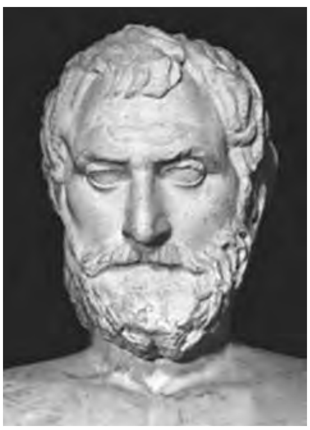
Bearing in mind that we are dealing with matters of fundamental importance, we need not apologize for the elementary nature of the following account. It should help refresh the mind on familiar matters, while opening some new flanks at the same time.

Constructing a Star Chart

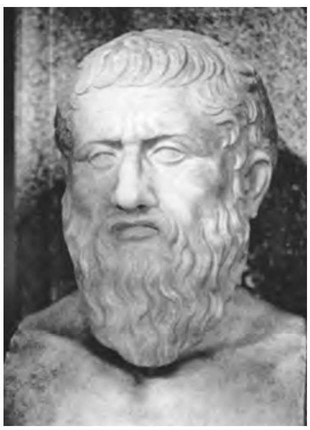
Imagine you are a prehistoric astronomer, attempting to produce a star chart on a clay tablet or papyrus sheet. You require that the chart should accurately represent the shapes of the familiar constellations of stars, and also the mutual orientations of the various constellations relative to each other, so that the chart can be used for navigational purposes.

As far as individual constellations are concerned, you find no difficulty drawing any one of them separately. You just naively transfer the image of what you see, *as if unchanged,* to the tablet. No problem? But, as you begin to map *larger* portions of the sky, adding more and more constellations to the chart, difficulties arise. The constellations don't fit together. You begin again, with another constellation as starting-point. Once again, things don't fit. Why? Although in each case you can specify the point at which the mapping process begins to break down, the underlying cause clearly lies *outside* the specifics of each attempt.

This problem embraces paradoxes of the sort any curious child will have observed. I stand up and look straight ahead at some point on the horizon. Now, I look to the right of that, and more to the right, and so on, until, by continuing my action of "looking to the right," I turn all the way around and come back to the original point . . . from the *left!* Or instead, if I start by looking straight ahead as before, and now look *up,* and keep turning my head in that "upward" direction further and further, I end up bending backward until I am moving my head *downward* toward the ground and seeing everything upside down!



Thales (625?-?547 B.C.)



EIRNS/Philip Ulanowsky

Plato (c. 428-348 B.C.)

This Pedagogical Exercise first appeared in December 1998.

(Let no one laugh off these simple paradoxes of linearity, who is not prepared, for example, to explain to any child or adult, how it can happen that the Earth can be in two different days, depending on the position on the Earth's surface, at one and the same moment in time.)

These sorts of paradoxes give rise to unavoidable, interwoven *periodicities* in our attempt to construct a star chart—as for example when I attempt to represent the observer's looking "to the right" and "upward" by motion "across" and "up" on the chart.

(At a more apparently "advanced" level, the same problems plague the Cartesian-like coordinate systems still used by astronomers to record the positions of the stars. To describe one such system in a perfunctory manner: Given any star, let "y" be its angular "height" above the horizon—i.e., the magnitude of angle from the position of the star "downward" to the point "directly below it" on the horizon—and "x" the angle along the horizon from that point to some chosen fixed point on the horizon. We might thus represent the position of any star by a point in the Cartesian plane, whose rectilinear coordinates are proportional to x and y, respectively. The resulting mapping, however, grossly distorts the shapes and angular relationships of the constellations, especially those in the vicinity of the overhead or zenith-point, where the mapping "explodes.")

This mere descriptive approach, however, falls short of identifying the underlying cause of the problem. In particular, it does not answer a crucial question which ought to pose itself to us: Does the difficulty arise only when we want to map *large portions* of the sky; or is it already present, albeit so far unnoticed, in the attempt to represent any *arbitrarily small* portion of the sky?

Spherical Bounding of The Universe

To progress further, we need to examine the internal characteristics of that action by which we, as ancient astronomers and navigators, are attempting to measure the Universe. The ancient astronomer makes a series of *star sightings,* measuring, in effect, the *rotation* from one direction in the sky to another. Imagine that a movable "pointing-rod" of fixed length is fixed at one end to a universal joint at our point of observation. Observe that the tip of that rod moves on a *spherical surface* whose center is the fixed pivot point, and whose radius is the rod's length. Imagine we were to construct a transparent spherical shell of that dimension around the center, and mark the shell at each position where the end of the rod points to a star. The result would be a spherical star-chart, whose markings would coincide *exactly* with the observed star positions when viewed from the center of the sphere (and only then).

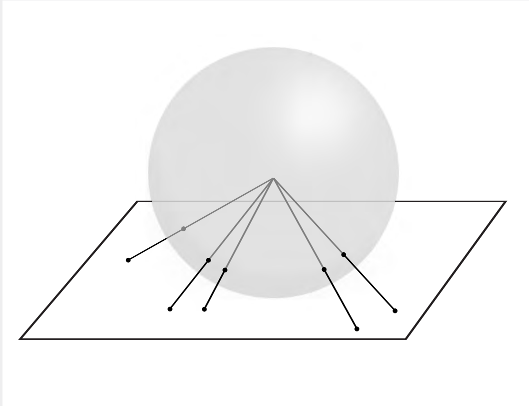


FIGURE 1. *Mapping the inside surface of a sphere onto a plane.*

We have demonstrated a *spherical bounding* of our action to measure the Universe! The sphere is not an object in the sky, but a determinate feature of our act of measurement: a representation of its underlying *ordering principle.* Does that make it arbitrary or "purely subjective"? By no means! This phase of astronomy is a necessary step in the self-development of the Universe, and thus an imbedded characteristic of the Universe itself.

It now appears, that the ancient astronomer's problem of drawing a star chart on a clay tablet or papyrus, is equivalent to the problem of mapping the inner surface of a sphere onto a plane surface. (Note: "Inner surface of a sphere" signifies—paradoxically enough—a *completely different* geometrical ordering principle, than the "outer surface." "Inner surface" signifies the ordering of the surface with respect to the spherical center only.)

There exist innumerable possible methods to attempt such a projection, each of which fails in a different way. The simplest is the method of central

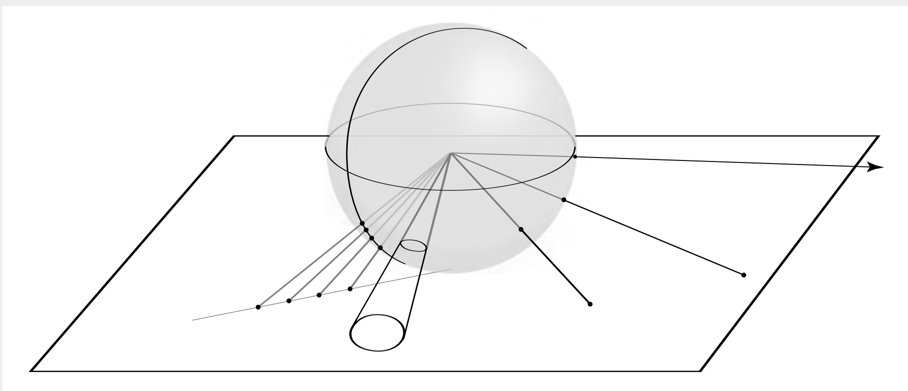


FIGURE 2. *Distortion in the projection is minimized in the polar region, but increases near the boundary of the hemisphere.*

FIGURE 3. *Constructing a map of the surface using congruent segments.*

projection onto a plane outside the sphere, defined as follows: For any locus on the inner spherical surface—corresponding to a pointing-direction from the center—prolong that direction outward until it intersects the plane [SEE Figure 1]. Readers should thoroughly investigate this species of projection with the help of a transparent plastic sphere and a suitable light source, noting several important characteristics.

For example: the action of simple rotation (e.g. of the pointing-rod) generates a *great circle* on the inner surface of the sphere; the projected image of a great circle, so constructed, produces the effect of a *straight line* on the plane surface. Encouraged by that result, examine the effect of the projection on various arrays of great circles. At the same time, observe that the projection maps only a *half* of the spherical surface, a hemisphere, onto the plane. The boundary of that hemisphere—a great circle whose location we can determine by cutting the sphere by a plane surface parallel to the projection-plane—defines a *singularity:* the mapping "blows up" when we approach that boundary circle. In the vicinity of the boundary, the projection introduces wild distortions relative to the relationships on the inner spherical surface. The least distortion apparent occurs farthest away from the boundary, in the "polar region" of the hemisphere [SEE Figure 2].

The "catastrophic" distortions near the boundary, and the circumstance, that only half of the sphere is mapped (or actually much less, if we want to avoid the worst distortions), suggests to our ancient astronomer the following tactic: Instead of trying to map the entire spherical surface (or night sky) at once, divide the surface into regular, congruent regions, and construct the "truest possible" mapping for each one [SEE Figure 3]. The combination of such sectoral charts would hopefully fit together to replace a single one. Note, that a complete set of central projections, of the sort we now envisage, corresponds to a *regular array of great circles* on the sphere, each constituting the singular boundary of the corresponding mapping.

Out of the corner of our mind's eye we might already have anticipated a new source of failure: The attempt to "fit" the mappings together at the edges of the chosen regions, will result in *discontinuities!*

We have entered into the domain governed by the five regular solids. We propose to explore that domain, from a new standpoint, in future pedagogical discussions. To finish this one, consider the following:

We saw, that in order to reduce the effect of distortion in each spherical mapping to a minimum, the portion of the spherical surface mapped, should be made as small as possible. But, how finely can the surface of the sphere be subdivided?

The characteristic of linear, planar, solid, or Cartesian geometry in general—a characteristics which distinguishes such hypothetical, "virtual" geometries from the real Universe—is the purported possibility of unlimited, self-similar subdivision, or "tiling," of space. Take a square in the plane, for example; by connecting the midpoints of the opposite sides, we can divide the square into four congruent subsquares, and so on *ad infinitum.* An analogous construction applies to any triangle. Similarly, a cube in so-called "solid geometry" can be divided into 8 (or any cubed number) of congruent cubes [SEE Figure 4].

What about the inner surface of the sphere? Take the division of the spherical surface into six congruent, curvilinear-square regions—i.e., a regular spherical cube. What happens when we try to subdivide those regions into smaller, congruent curvilinear squares? What happens for the division of the spherical surface, defined by the regular octahedron, and the other regular solids? What is the *common source* of the barrier to further subdivision?

—Jonathan Tennenbaum

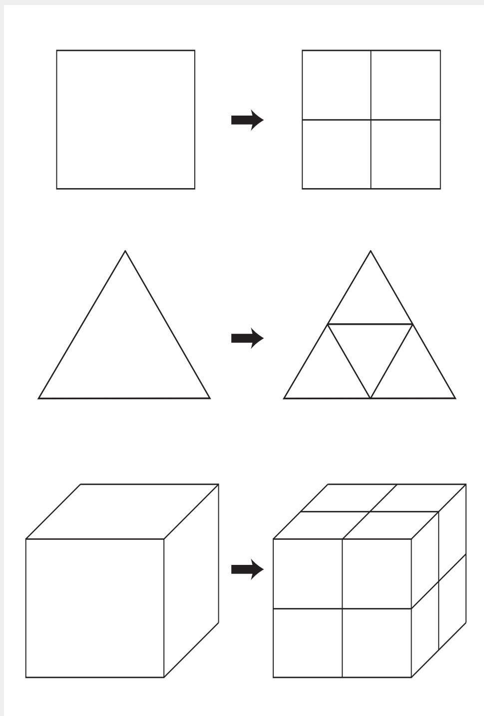


FIGURE 4. *Tiling a square, a triangle, and a cube.*