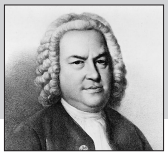


On the Subject of Strategic Method

Bach and Kepler: The Polyphonic Character Of Truthful Thinking

by Jonathan Tennenbaum

In my remarks I will address the fundamental point that Dino de Paoli has taken up, but from a completely different angle.

Twenty-five hundred years ago, the Chinese philosopher Confucius said he could determine the political and moral character of a nation, just from its music. Imagine if Confucius were here today, and he would hear the typical kind of music which 95 percent of young people today are listening to, often all day long.

What would Confucius say about this? Surely he would say: "Uh, oh! Here is a doomed dynasty, a dynasty which is in a late stage of inner collapse." And he would be right. Although we do not exactly have a dynasty in the traditional Chinese sense, we do have a *dynasty of ideas,* of commonly accepted opinions and attitudes, which is collapsing before our eyes. None of the opinions and beliefs, which typify public and private life today, are going to last very long. They will soon be swept away by the onrush of perhaps the greatest crisis in human history.

But what if we were to ask Confucius, how do you *know,* Mr. Confucius, from listening to the music, that the present dynasty is doomed? How can you make such a forecast? He would certainly answer: "Because, from the music I can determine the character of the mental processes prevalent in a society, and above all the prevailing conception of Man. Those tell me whether a civilization will develop, or collapse."

Confucius already knew the bare kernel of the method, which was later practiced, with great success, by Johannes Kepler and Carl Friedrich Gauss, in their discovery of a missing planet in the solar system. It is the method of *characteristics,* as Gottfried Wilhelm Leibniz understood the term, which is also equivalent to the conception of a general, anti-Euclidean physical geometry, developed by Bernhard Riemann as a continuation of the work of Kepler, Leibniz, and Gauss. It is also the method of well-tempered counterpoint, as developed by Johann Sebastian Bach. It is the opposite of information theory and cybernetics.

To bring this out most forcefully, I want to focus on a crucial historical branching point, when the issue of method in musical composition—and implicitly in all of human knowledge—took a particularly drastic and pedagogically useful form.

Bach *vs.* Rameau

In 1722, Johann Sebastian Bach launched a musical revolution, with his publication of the first book of *The Well-Tempered Clavier,* containing 24 Preludes and Fugues in all keys, and demonstrating for the first time the full potentialities of well-tempered, vocal-based counterpoint. Bach did not add any commentary or theoretical analysis; for him, music was a fully developed language, and

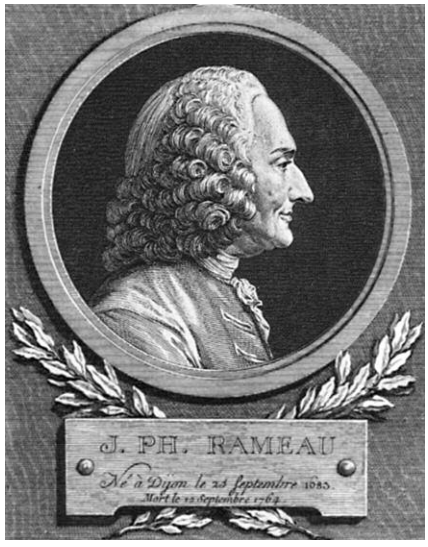
Bach said everything he wanted to say, without any ambiguity, in the music itself.

But in that same year, the French organist Jean-Philippe Rameau published his *Treatise on Harmony,* claiming to have discovered the *fundamental law* of all musical harmony and composition, a law based on mathematics. According to Rameau, the principle and basis of all music is located in what he called "*la basse fondamentale*." Rameau promises, by reducing the successions of notes in a piece of music to the single line formed by the "fundamental bass," to make the study of music much easier:

> [My] book contains a special method for learning how to compose music in a very short time. This has already been tested.

Rameau was soon hailed in France as the "Isaac Newton of Music." His fame spread rapidly throughout Europe, and his book became the single most influential writing on the theory of music up to this day. Rameau was the basis of Helmholtz's later work, which in turn was taken as authoritative for all the Twentieth-century teaching of music theory. If you study musical harmony at virtually any music conservatory or school today, what you will get, essentially, is Rameau.

From the very beginning, Rameau's theory was strongly promoted by the encyclopaedist d'Alembert, Voltaire, and the same British-Venetian salons that sponsored the so-called Enlightenment, and promoted the cult of Isaac Newton in France and on the continent generally. The effect in France was so enormous that, thirty years after the publication of Rameau's *Treatise,* Jean-Jacques Rousseau, famous as a music critic, described the situation with the following words:



The Granger Collection

Jean-Philippe Rameau, opponent of the Bach school of vocal polyphony.

> The study of composition, which used to require about twenty years, now can be completed in a couple of months; musicians are devouring the theories of Rameau, and the number of students has multiplied. . . . France has been inundated by bad music and bad musicians; everybody thinks he has understood the finesses of art before having learned as much as the rudiments; and everybody tries to invent new harmonies before having trained his ear to distinguish between right and wrong ones.

Rameau's popularity practically meant the end of rigorous musical composition in France and a great part of Europe. Bad music, particularly opera, became a kind of plague, similar to the rock and pop music of our days. Rameau's work was spread into Germany by Marpurg and others, against the violent resistance of the Bach school, who immediately recognized that Rameau's ideas constituted a deadly assault on the whole Renaissance principle of composition, which had been based on *vocal polyphony.* The ensuing history of music has been a *war* between the continuators of the Bach tradition, and the followers of Rameau—which is essentially the same thing as the *Romantic* school. Mozart and Beethoven, for example, were still rigorously trained in Bach. But by the time of Brahms, the Bach tradition had been uprooted from most musical education, to the point that Brahms himself complained bitterly to his student Jenner, that he, Brahms, had suffered enormously from "bad textbooks" and had to learn everything over again.

Rameau's Theory *Per Se*

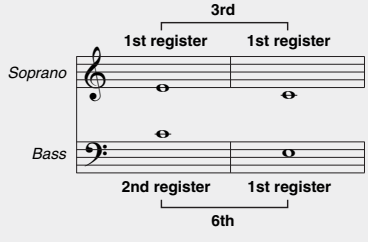
Now, let us briefly look at Rameau's theory and what was wrong with it.

Rameau starts out, very characteristically, by declaring: "Music is the science of sounds; therefore, sound is the main object of music."

Wait a minute! What about the human mind, what about the *ideas* which music is supposed to express? What Rameau says would be like saying, that the principles of poetry can be deduced from the properties of words. The notion, that a musical composition has a *meaning,* plays no role in Rameau's work. Later, in his last musical treatise, Rameau is even more explicit:

> We finally possess the principle of this profound knowledge which

FIGURE 1. *Transformation in intervals.*



The interval of a 3rd sung by a soprano, contains no register-shift, whereas that interval's inversion, when sung by a bass, contains a shift from the 2nd to 1st register.

will render immortal the glory of mathematics and mathematicians. This principle is in the vibrating body.

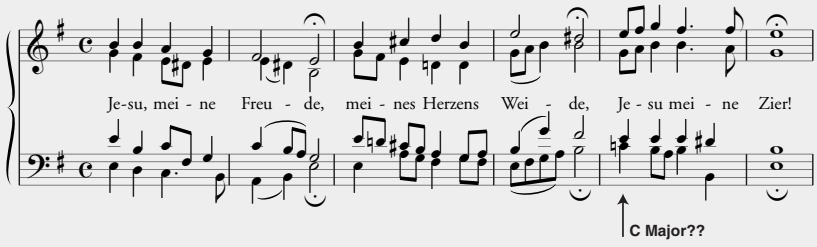
So, for Rameau, it is sense perception and the vibrating body which determine what music is. The human mind plays no role! This is exactly what you find today in a discotheque: vibrating bodies, but no thinking human beings! The only additional factor is what Rameau calls "taste" (*gout*) and "license," which are unexplainable and arbitrary.

Rameau's theory pays no attention to the principles of *vocal counterpoint,* which had been at the center of the development of music since the Renaissance. Instead, he presents a static notion of *vertical harmony* on the basis of *chords,* or momentary constellations of notes that happen to be sounded at the same time.

Start with a vibrating string of some length, which produces some tone. Divide it into two, three, four, and five parts, which are the simplest arithmetical divisions. The chords of lengths 1/2, 1/3, 1/4, and 1/5 produce tones, which together with the original tone form a *chord.* Rameau considers this chord the foundation of music. I will play it on the piano [demonstration]. The upper tones coincide with the so-called "overtone series," experimentally established by Sauveur and others around the same time.

Now Rameau argues that the relationship of an *octave*—corresponding to division of a string by two—is a kind of *identity.* We hear any tone, and its upper or lower octave, as being essentially the same note. As a result, according to Rameau, we can replace any note in the fundamental chord by its octave, and we will get a chord which is harmonically equivalent. By such rearrangements, we get a whole series of chords, known as inversions, which are all derived from the same bass tone, or "*basse fondamentale,*" and which Rameau regards as essentially equivalent.

FIGURE 2. *Opening of J.S. Bach's motet "Jesu, meine Freude."*



But if you take the standpoint of vocal polyphony, octaves are not at all equivalent: Replacing a note by its octave transforms all the relationships among the different voices! Firstly, I end up in a different *register* of the voice, or even in a *different voice.* Secondly, I transform the *intervals* between the voices. For example: a third becomes a sixth, and a fifth becomes a fourth, and vice-versa [SEE Figure 1].

This sort of *transformation* of intervals, known as *inversion,* was of course known long before Rameau, as a principle of *development* in vocal-based counterpoint. But for Rameau, there is no real change, because the fundamental bass remains the same.

In fact, Rameau thinks of music as a sequence of individual sound effects. He has no conception of a process of transformation, like someone who speaks only in nouns, without verbs.

Let me show you a simple example to see how completely incompetent this so-called theory is: a very simple chorale which Bach uses at the opening and closing of his motet, *Jesu, meine Freude.* In Figure 2, in the measure marked with an arrow, we see, on the first syllable of the word "Jesu," what Rameau would identify as a perfectly simple consonant chord on C. Perfectly consonant, that is, when we play it or sing it *in isolation.* Similarly, Rameau would identify the notes just before it, at the end of the preceding measure, as a perfect consonant chord on B-natural.

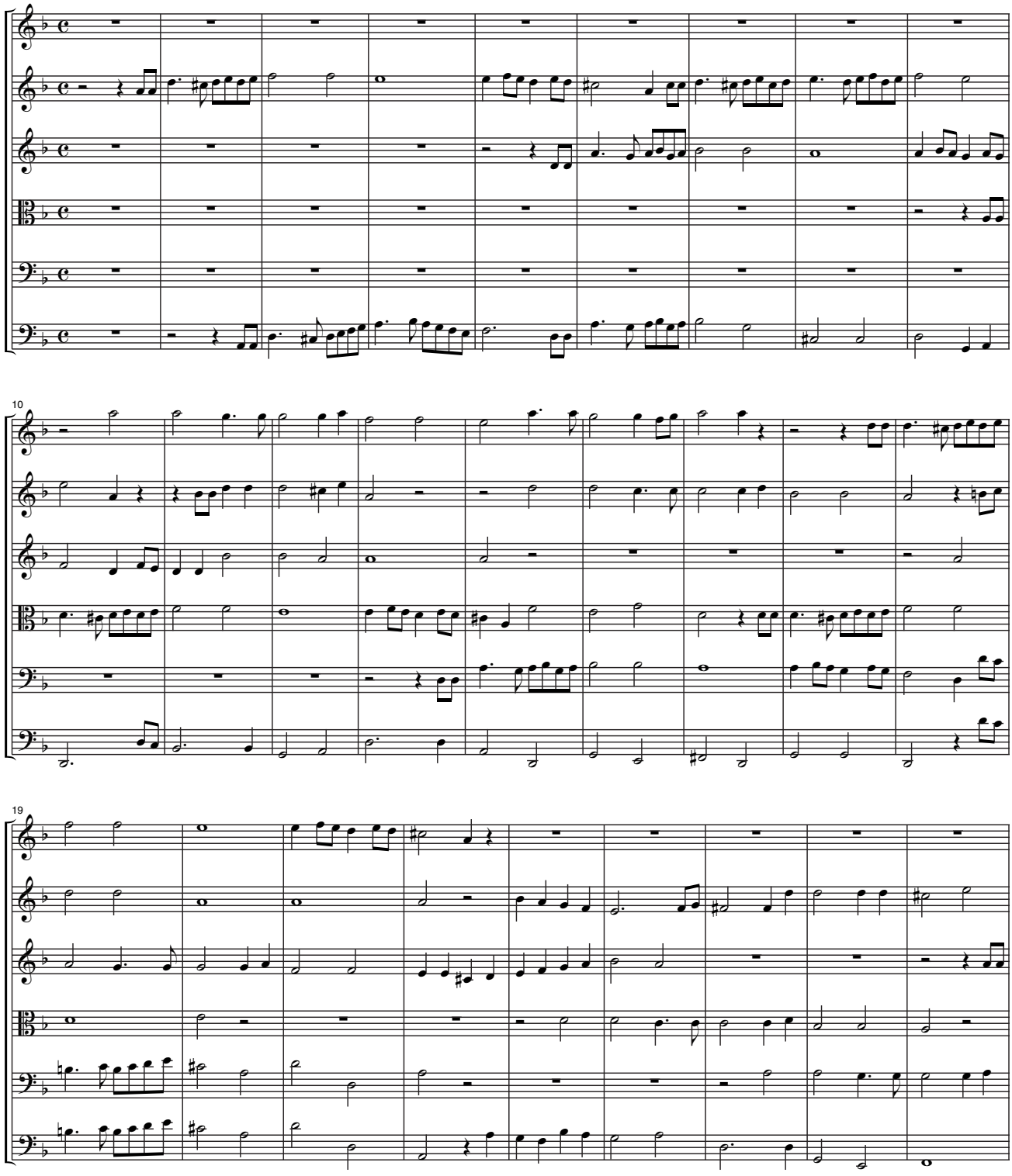
But, if I play the two in succession, as fundamental chords in Rameau's sense, I get just nonsense. Whereas, in the context of the actual chorale, the moment where the supposed "chord" of C sounds, is a moment of great tension, a kind of *dissonance,* which is resolved by the development on the following words, "meine Zier!"

Examples like this demonstrate some obvious points, refuting Rameau's whole approach:

First, human beings don't hear music as a succession of chords or sound effects, but rather as a process of transformations. It is not the sound of a momentary constellation of notes that determines, for example, whether we hear a given moment as consonant or dissonant, but rather the context, the *process* subsuming those notes.

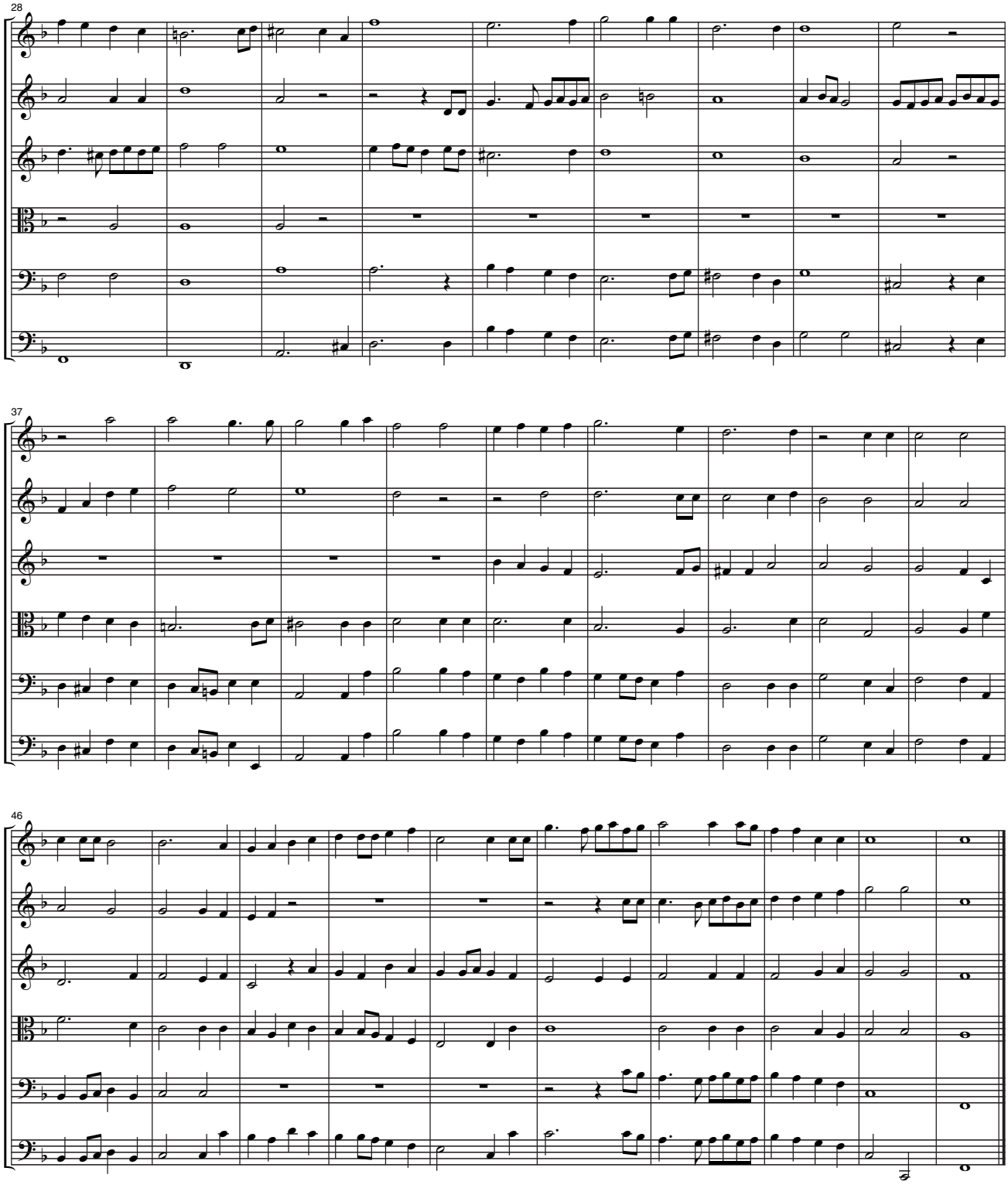
Second, each of the voices in Bach's chorale has its own characteristics, its own *willful motion;* so that each moment of such a polyphonic composition is like an historical event, in which various different processes intersect and interact with each other. The dramatic moment at the word "Jesu" is especially connected with the upward motion of the bass,

FIGURE 3. *So-called "fugue" by Jean-Philippe Rameau.*



continued on following page

Figure 3 (continued)



which begins in the preceding measure, and is continued by the soprano and alto voices; and with Bach's special use of the half-tone intervals B♮-C-B♮ and F♯-G-F♯ in the various registers, which lends this chorale (and the whole motet), a specific *character.*

How To Write Bad Music

These sorts of things were well known to any competent composer in Rameau's time, but he just ignores them and goes on to formulate his instructions on how to write music. First, you invent a fundamental bass, according to your own good or bad taste. Rameau advises composers who have no taste, to just choose the fundamental bass at random! Next, you just write down the corresponding chords, choosing whatever distribution of intervals suits your mood. In some places you can omit the fundamental note, and use one of the other notes of the chord as the bass note (so-called chordal inversion). Finally, you can add some dissonances according to your arbitrary "taste." Congratulations! You are now a bad composer!

In fact, Robert Schumann criticized a typical product of the Rameau school, the Waverly Overture by Hector Berlioz, in the following words:

> Often it is only a series of empty sound-effects, of lumps of chords thrown together, which seems to determine the character of the piece. . . . He shines like a jet of water, but he also leaves a stinky smell of sulfur behind him; he puts forward great propositions and truths, only to fall thereafter into schoolboy-like babbling.

Friedrich Chopin made a very similar criticism of Berlioz, in a discussion reported by Eugène Delacroix:

> It has become customary now to learn chords ahead of counterpoint, which means, ahead of the sequences of notes by which the chords are formed. Berlioz simply sets down the chords and fills the interstices as best he can.

FIGURE 4. *Opening of Fugue XXII from Part I of J.S. Bach's "The Well-Tempered Clavier."*

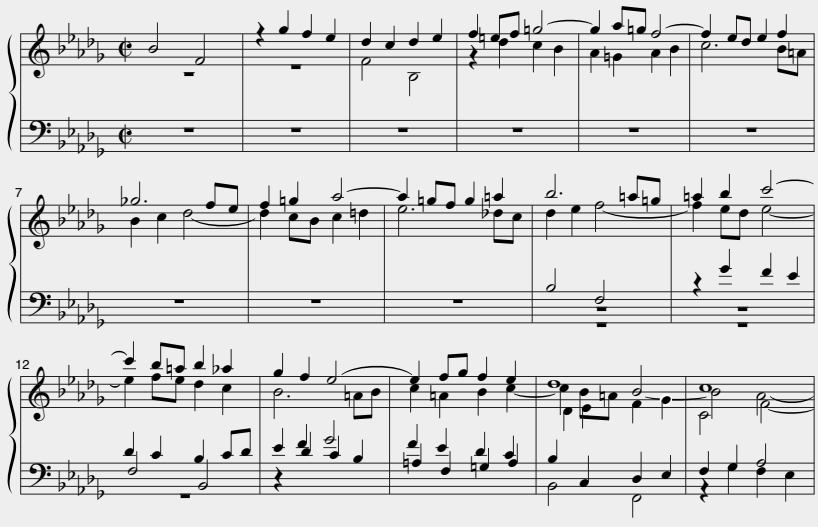
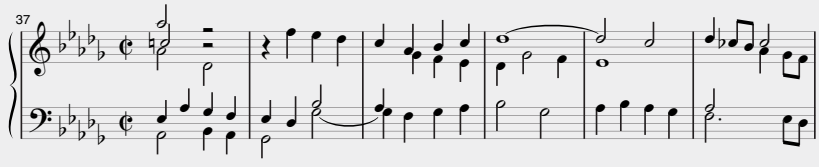


FIGURE 5. *Passage from Fugue XXII of Part I of J.S. Bach's "The Well-Tempered Clavier."*



A Revealing Comparison

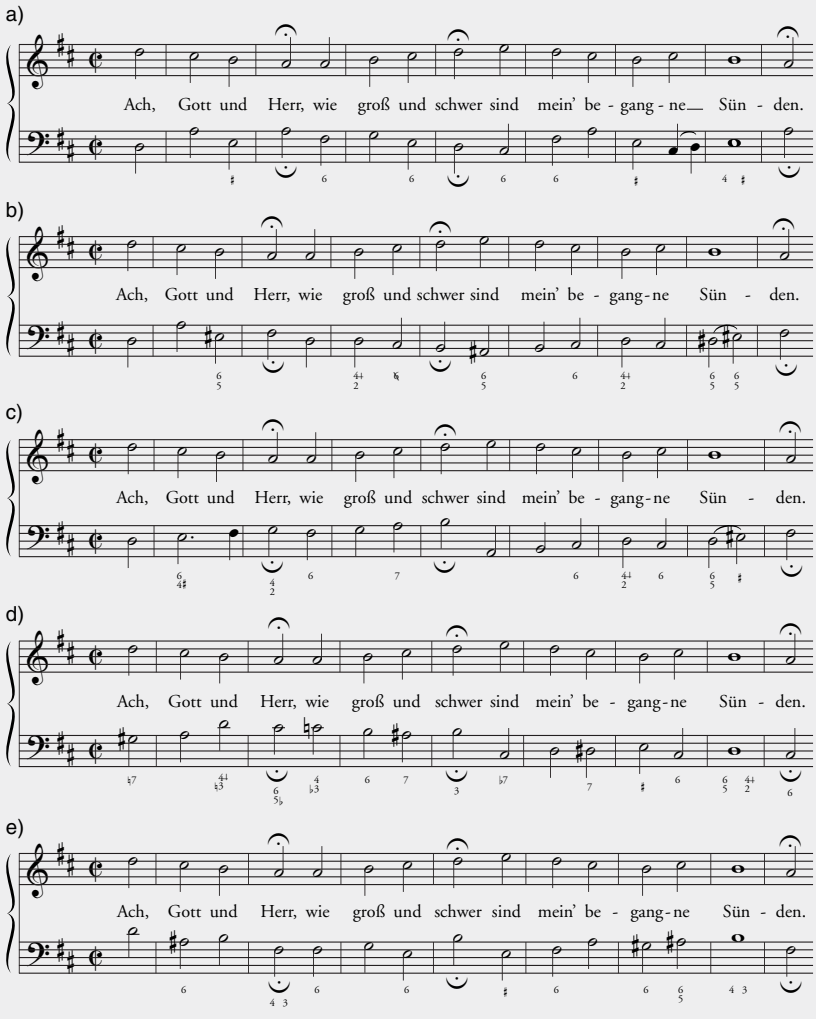
Let's hear the beginning of a piece by Rameau himself, which he uses as an example in his famous *Treatise on Harmony* of 1722. Rameau calls this a fugue, but it is so, at best, only in a formal sense. And then, let's compare that with the opening of a real fugue, Bach's B-minor fugue from *The Well-Tempered Clavier,* written in the same year, and which Beethoven later studied particularly carefully when he was working on his *Hammerklavier* sonata. I am playing the Rameau fugue on a computer, which is perhaps appropriate to the quality of his method [SEE Figure 3].

At first hearing, it sounds nice, and you can be fooled by Rameau, who is skillful at creating harmonic sound-effects and putting short counterpoint imitations in. But it is not a fugue, because there is no *dialogue,* no tension between the *voices.* Not surprising, because in Rameau's theory the voices have no real existence; they are essentially devised after the basic outline of the composition has been finished, to fill in the spaces between the chords, as Chopin pointed out.

Now let me play just the opening of Bach's fugue [SEE Figure 4].

Could there be any greater differ-

FIGURE 6. *Johann Philipp Kirnberger, five exercises on the same chorale melody.*



ence? Bach's fugues are *dramas:* The voices enter like persons in a Shakespeare or Schiller play; each intervention *changes* the process, and the composition drives forward, with enormous pungency, tension, and force, from one crucial conjuncture to the next. This is a typical example of a Keplerian curved space-time, as we shall see at the end of my presentation.

Indeed, Bach's polyphony involves a special sort of harmony, not explicable by Rameau's sort of theory. To emphasize this, I want to play a short passage from later on in the same B-minor fugue [SEE Figure 5].

This passage is completely anomalous to either Rameau's theory, or the rules of textbook counterpoint. Looked at abstractly or statically, the voices make a jarring series of dissonances, without formal resolution. But we do not hear their motion as dissonant! The reason it works, cannot be deduced from the notes *per se,* but lies in the *intelligible idea* which is guiding the voices. That is why they seem to progress without difficulty toward their respective destinations, like planets travelling along a set of least-action harmonic "orbits," moving in a special curved space-time, which has been created by the composer in the mind of the listener, not by some fixed *a priori* rules. As a result, Bach is constantly able to do "impossible" things, breaking the rules in a truthful way. In fact, Bach's student Kirnberger reported:

> The great, late J. Seb. Bach used to say: It must be possible to do everything; he never wanted to hear that something was impossible.

Bach's Platonic Polyphony

Now, the key to Bach's special use of harmony, is the way each voice *changes* the way each other voice is heard. Let me do a very simple pedagogical demonstration of this, which was made by Bach's student Kirnberger [SEE Figure 6]. He took the opening phrases of a very simple chorale melody: "Ach Gott und Herr, wie gross und schwer sind mein' begangne Sünden," and wrote 26 different bass lines to it, each bringing out a different *sense* and coloring in the original melody. In doing so, Kirnberger emphasized the conception of harmony, not as a matter of chords, but as a means of *contrapuntal development.* We will just do five of them to give you an idea of this. (Note that this is not intended to be a real piece of music, but only a laboratory demonstration.)

Naturally we can also start with a bass voice, and, by adding different soprano voices, *transform* the meaning of the bass. Thus a real dialogue and drama between the voices becomes possible. Listen, from this standpoint, to the opening of the fifth fugue of Bach's *The Art of the Fugue,* in which the dialogue is especially clear. In this fugue the subject appears together with its *mirror inversion,* thereby making it possible to generate a new set of cross-voices [SEE Figure 7].

A useful reflection of Bach's con-

ception is contained in the first (and I think, best) biography of Bach, written by the music director of Göttingen University, Nikolaus Forkel, based on discussions and correspondence with Carl Philipp Emanuel Bach, Kirnberger, and others of J.S. Bach's students. Forkel characterized Bach's use of harmony in the following way:

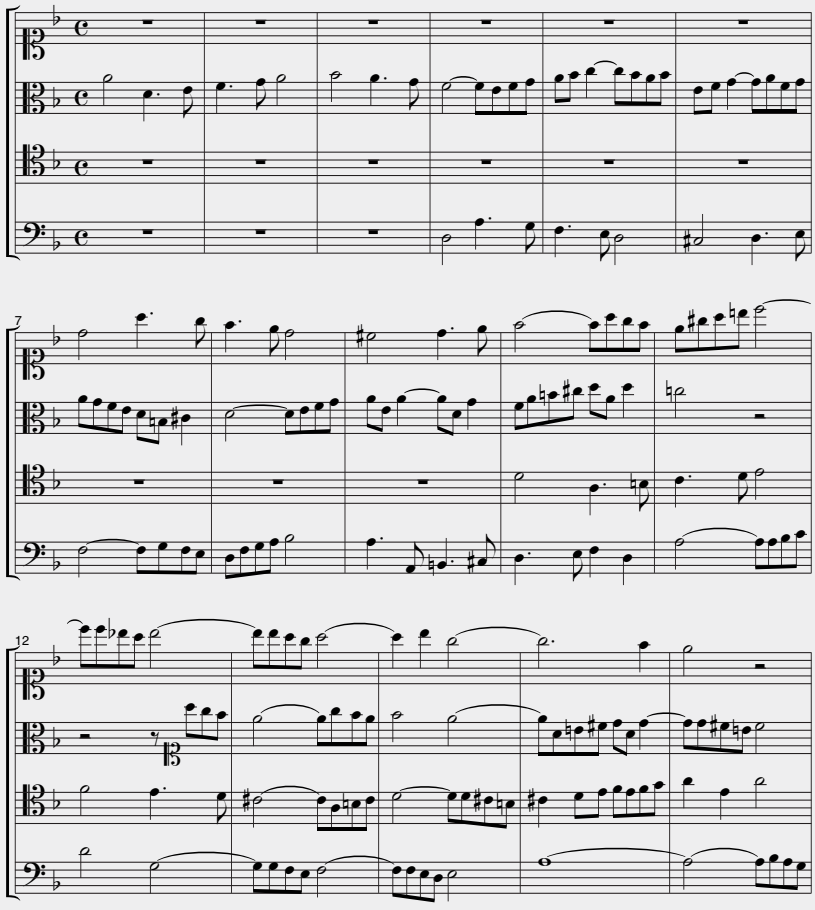
> He considered music entirely as a language, and a composer as a poet. . . . [But] so long as the language of music has only simple melodies, or only successive connection of musical tones, it must still be considered poor. . . . Very different is the case, when two melodies are so interwoven with each other that they, as it were, converse together, like two persons of the same rank and equally well informed. . . . This sort of union of two melodies gives rise to new combinations of tones and consequently to an increase in the variety of musical expressions. When more voices are added, and interwoven with each in the same free and independent manner, the wealth of musical expression increases still further. . . . Harmony must thus be understood not simply as the accompaniment of a simple melody, but as a real means to increase . . . the wealth of our musical language.

Here Forkel is explaining what has since become known as the "cross-voice" principle: New musical ideas are generated, so-to-speak, *between* the voices. So it is, also, in a dialogue of Plato, or a drama of Aeschylus, Shakespeare, or Schiller.

Truth in Music

So much for the musical examples. You will hear more tonight in the concert and tomorrow morning in the panel on Bach. But now I want to start some trouble. In pursuing the diametrical opposition between the methods of Rameau and his followers (the Romantic school) and Bach,

FIGURE 7. *Opening of Contrapunctus V from J.S. Bach's "The Art of the Fugue."*



we have arrived at a point which is very upsetting to many people.

A: Wait a minute! You and Forkel talk about a musical language, a dialogue, and so forth. But, what is the dialogue about? What are the voices in a Bach fugue *saying* to each other? Can you express it in words?

B: No.

A: Aha, it is a different sort of information.

B: No. Not information.

A: Some message coded in symbolic form?

B: No!

A: Then you are talking about *feelings*. The voices are expressing pure feelings.

B: No!

A: Then, surely you do not really mean to say that the voices in Bach's fugues are making an actual dialogue. Surely Bach is just *imitating* a conversation, just like some composers imitate birds or scenes in nature in their compositions.

B: No. It *is* a dialogue. Classical polyphony is a real language, and the compositions of Bach and his school, up to Brahms, have a definite *meaning*.

A: I think you are interpreting too much into the intentions of composers. After all, art is purely subjective, and creativity is something mystical. Especially when you get to Brahms, which is the Romantic period.

B: No, nonsense! Brahms was a passionate anti-Romantic, like all the

great composers of the Bach school. Just read the book his student Jenner wrote on how Brahms taught him. Brahms was a fanatic on rigor. For example, when Brahms and Jenner were discussing the problems of writing variations on a theme. Brahms advised Jenner: "The fewer variations the better; but then they must say everything that is to be said."

From this and a thousand similar remarks, it is absolutely clear, that when Bach, Mozart, Beethoven, or Brahms wrote a piece of music, they knew *exactly* what they wanted to say.

A: So we are back to my original question. What is the *meaning* of a Bach fugue? I listen to it again and again and look at the score, but I can't figure it out.

B: The problem is, you are looking in the wrong place. The meaning is *not* in the notes.

A: What? If the meaning is not in the notes, where is it then?

B: In your *mind.* If you have listened to a piece and it was performed properly, then the *idea* Bach is expressing with the help of the dialogue, has already been generated inside your *mind.*

A: That is absurd! If the idea were already in my mind, I would not be asking *you* for it!

B: It *is* in your mind, but you don't recognize it, because you are looking for a literal sort of meaning or interpretation. But the meaning is a *thought-object,* which Forkel and Kirnberger, for example, had in mind when they talked about the *character* of a composition, and which derives from the particular method or hypothesis which Bach chooses in generating and resolving the *paradoxes* in the musical dialogue. In fact, not only is Bach expressing musical ideas, but his compositions are also *true.*

A: This is too much for me. How

The Granger Collection

Johannes Kepler

can you claim that a musical composition expresses truth, if you can't even tell me what it is saying? After all, "truth" means to be in agreement with the *facts.*

B: But what about a *thinking process?* Cannot a thinking process be either truthful or fallacious? Regardless of so-called facts?

A: Give me an example.

B: Well, for example, you would agree that the world is in the midst of a gigantic economic, financial, political, and moral crisis?

A: Yes, that is why I am here.

B: But, what about your friends and neighbors, who have the same facts as you, but don't see any big crisis at all? You see the destruction of the economy, and they see prosperity and growth. What is the reason for that?

A: Something is wrong with their heads.

B: Exactly! Their thinking processes are not *truthful.* That is exactly what Confucius could tell, just by listening to today's popular music. In the same way, the compositions of Rameau are false, they are a facade; whereas Bach's compositions represent truthful thinking.

Bach and Kepler

A: But now something else bothers me. You pointed out that Bach's composition does not follow formal rules, of the sort that Rameau and others tried to define. In fact, Bach constantly breaks the rules. But apart from agreement with facts or with formal logical or other rules, how can you *know* whether Bach's or anyone's thinking processes are truthful? Aren't you opening up the door to purely subjective opinions?

B: Well, the question of truthfulness cannot be addressed simply within music *per se.* Ultimately, it is a matter of physics, or more precisely, of man's active relationship to the universe as a whole. What processes of the mind lead to an increasing power of mankind, *per capita,* over the physical universe? To the extent we can identify, in our own minds, the characteristic of such creative processes of generation, assimilation, and application of valid scientific discoveries, we can know the truthfulness of our own mental processes. At the same time, by knowing creative Reason, we can know the principle of creation itself, in the only way we could possibly know it! This is why the development of music, since the very beginning of human culture, has been inseparable from *astronomy.*

A: Astronomy?

B: In fact, there is no doubt that Johann Sebastian Bach's anti-Rameauvian revolution in music, was based directly on the work of Johannes Kepler—exactly the same work which led to the subsequent development of an anti-Euclidean physical geometry by Leibniz, Kästner, Gauss, and Riemann.

A: What could Bach's compositional method have to do with anti-Euclidean geometry?

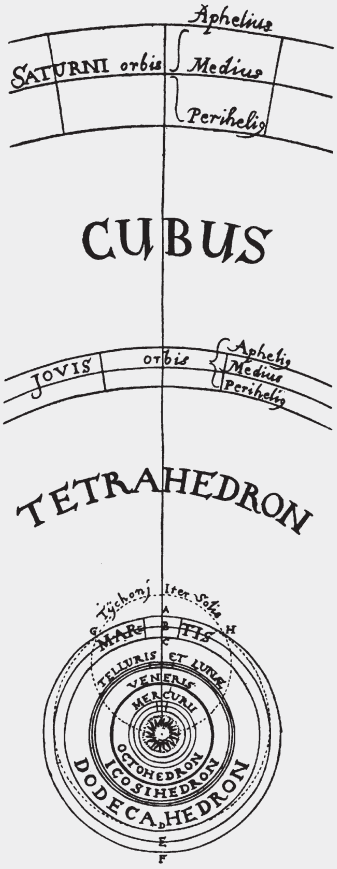
B: Everything. In fact, Andreas Werckmeister, probably one of the greatest influences on the young J.S.

Bach, explicitly stated that it was Johannes Kepler who provided the astronomical proof for the well-tempered system of vocal polyphony. Werckmeister wrote extensively about this, particularly in a remarkable book entitled *Musikalisches Paradoxon-Discours* (*Discourse on Musical Paradox*), in 1707. So, we can be quite sure that Bach's 1722 *Well-Tempered Clavier* is a musical elaboration and celebration of Kepler's astronomical discoveries.

Incidentally, Abraham Kästner, the teacher who brought the key issues of anti-Euclidean geometry and the work of Kepler to the attention of the young Carl Gauss in Göttingen, came from Leipzig, where he taught at the university at the same time that J.S. Bach was the main musical figure in the city. A fellow teacher at the Leipzig Thomasschule together with Bach, Johann Winkler, organized the revolutionary experiments on electricity, which made Winkler famous as the "German Benjamin Franklin." Winkler also performed in Bach's cantatas. So, we can be sure that all of these matters were being discussed in Bach's Leipzig circles in the last fifteen years of Bach's life.

A: I still don't really see the connection with music.

B: Very simple. First, you should remember that the conception of a *curved* physical space-time is very old. It begins already with the discovery, made by the earliest astronomers of prehistoric times, that our naive sense-perception distorts the universe in a very specific way, leading to the false appearance that the world consists of a collection of discrete objects of an infinitely extended space, or that the elementary form of action is straight-line motion. On the contrary, the ancient astronomers knew that the geometry of the world is not *flat* in that way, but rather essentially *spherical;* and that this is reflected in the discovery



Kepler's determination of the harmonic ordering of the solar system, from his 'New Astronomy.'

of a growing array of astronomical *cycles* governing the motion of the planets and other phenomena. This led into the study of *harmonic proportions* of the heavens and of living organisms, as reported in Plato's *Timaeus;* and the notion, that our universe is *harmonically ordered* as a totality. That is the first form of anti-Euclidean geometry—that is, a geometry which is not based on deductive axioms, but on the discovery of *physical principles.*

A: Aha. So Bach's polyphony is based on this notion of a curved universe, as opposed to the flat thinking of Rameau and his followers.

B: Exactly. But there is more: You have Kepler coming along beginning in the 1590's, and reworking the whole question; as a first step, Kepler had to eliminate the corrupting influence of Ptolemy's formalist methods (the so-called epicycle method), which had blocked fundamental progress in astronomy for 1,500 years, since the Roman Empire's suppression of Plato.

A: So Rameau was really a successor of Ptolemy.

B: Absolutely. Just like the "information theory" freaks today, who are practicing pure Ptolemicism.

A: So, what did Kepler accomplish?

B: Briefly, in his first work, *Mysterium Cosmographicum,* Kepler developed a much-improved form of the Platonic hypothesis, that our solar system is pervasively *shaped,* in all its features, by a *unique principle*—a *physical* principle which is reflected, in visual-geometric terms, in the existence of exactly five regular solids in visual space, all of which are derived from a single one (the dodecahedron) in the manner Leonardo da Vinci's teacher Luca Pacioli demonstrated in his book on *The Divine Proportion.*

Kepler drew two very crucial conclusions: First, that the *origin* of the harmonic proportions, found in the forms of living organisms, the motions of the planets as well as in musical system, does not lie in self-evident properties of whole numbers (as the cabbalist Fludd tried to claim), but rather in an underlying *physical-geometry* of the universe as a whole. Second, the pervasive presence of the Golden-Mean-related harmonic proportions in the solar system—proportions otherwise found only in living processes and their products—suggests that the solar system had to be seen, not as a fixed entity, but as an *evolving process.*

A: Aha!

B: It was from this standpoint, informed particularly by the work of Nicolaus of Cusa, that Kepler turned to examine the discrepancies in the

orbital values, relative to a simple-geometrical determination in terms of the regular solids. He inferred the existence of an *additional,* yet-undiscovered physical principle underlying the organization of the solar system, and focussed his attention on the anomalies in the available astronomical data. After many years of work, Kepler published his *Nova Astronomia* (*New Astronomy*), demonstrating the elliptical orbit of Mars and establishing a new physical principle of *non-constant curvature,* which revolutionized all of science.

A: And Bach's revolution in well-tempered polyphony flows from that?

B: Yes, but not until we have solved the problem, to which Kepler addressed his final work on this subject, his 1619 *Harmonices Mundi* (*Harmony of the World*): How to *integrate* the principle of non-constant curvature, with the harmonic principle he had established twenty years earlier, in his *Mysterium Cosmographicum.* The problem is very simple: Instead of a solar system governed by simple astronomical *cycles,* we now have a process which is *changing its characteristics* from moment to moment, within every interval. What, therefore, is the higher *characteristic of change,* which subsumes the *evolving characteristics* of the system?

A: Like the motion of Mars on its elliptical orbit, where the velocity and the curvature of the pathway are different at every point?

B: Yes, but more than that, Kepler is addressing the *entire solar system* as a single process—in which, for example, each planet constantly reacts to the existence of all the other planets. Kepler demonstrated, that the harmonic values of any pair of planetary orbits—their minimum and maximum angular velocities as seen from the sun—form *musical* intervals. However, those musical intervals do not constitute a simple harmonic series, like Rameau's fundamental chords; nor do they fit together unambiguously into a single musical scale or tonality. The solar system does not work that way; it is genuinely polyphonic, and it generates *dissonances* in a lawful manner.

Carl F. Gauss

A: Did Kepler really say that?

B: Not only did he say it, but he called on the musicians of his day to assimilate his discovery:

> Follow me, you musicians of today, and judge for yourself. According to the principles of your art, which were still unknown to the ancients. . . . Through your polyphonic melodies, through your ears, the human spirit—the beloved child of the divine Creator—Nature has revealed her inner Essence. . . . The planetary motions are thus nothing else than a continuing, polyphonic music (perceived by the mind, not the ear); a music, which progresses through dissonant tensions, as if by syncopations and cadences (as Man uses these, in imitation of those natural dissonances), toward certain predetermined points of completion; and by doing so, sets its various marks onto the immeasurable expanse of time.

The fact, that the orbital values do not fit into a single, simple harmonic series, has two profound implications: First, from the standpoint of musical polyphony, we require a *well-tempered system,* because each pair of values must be "heard" not as an isolated interval, but in potential relation to all the *other* intervals in the system. Second, and more important: We live in a universe which cannot be reduced in a *deductive* manner to a single principle, as Newton claimed to do with his universal gravitation (itself actually a discovery lifted from Kepler). Rather, human knowledge develops as a growing family of physical principles, such that the discovery of each new principle modifies or tempers all the others. There is a higher characteristic or principle of discovery governing this process, but it is accessible only to the creative processes of the mind, and cannot be represented or communicated in any formal manner.

Finally, I should mention that at the end of his *Harmonices Mundi,* Kepler speaks of dissonances in the array of planetary intervals, as pointing to the possible existence of a "missing planet" between Mars and Jupiter—a possibility he had already discussed in his *Mysterium Cosmographicum* twenty years earlier. Less than a century later, the young Carl Friedrich Gauss, working on the basis of the overall *characteristics* of the solar system, demonstrated by Kepler, determined that the orbit of the asteroid Ceres—whose discovery Gauss himself had made possible—lay exactly in the orbital region Kepler had predicted!

In this way, the truthfulness of Kepler's—and Bach's—polyphony was established, to the glory of God and the delight of the human mind.