

So, when you look at this exploration of Eratosthenes' student Maui—the “eyes of the dragon,” so to speak—you say, “Well, why did 1,723 years pass between the discovery, or declaration of discovery of South America by the navigator Maui, and the similar discovery, the similar voyage of exploration, conducted by Columbus?” Why did 1,720-odd years have to pass?

*Because of a great degeneration of culture. And there-*

fore, when we look at European civilization today, and its legacy, that is the first measuring rod you must apply to understand the history of European civilization. You have to account for a crucial fact: that, from the time of the rise of the Romans until the Renaissance, European civilization was in a process of moral and intellectual degeneration. And we have not fully corrected that error yet.

## The Sieve of Eratosthenes

One of Eratosthenes' most important discoveries, was his unique method for finding the prime numbers, now known as the “Sieve of Eratosthenes.” Among the whole numbers, there exist unique integers known as prime numbers, which are distinguished by the property that they are indivisible by any other number except themselves and 1. Thus, 2, 3, 5, 7, and 11 are all examples of prime numbers. Numbers such as 8, 9, and 10 can be evenly divided by other integers and are thus called composite.

Eratosthenes' method of finding the primes functions exactly like a sieve, in which the composite numbers fall through the “mesh,” and the prime numbers remain. The “mesh” in this case, is the ordering principle by which the composite numbers are generated from the primes. To this day, Eratosthenes' method is essentially the only one for finding the prime numbers. More important, his approach of investigating numbers in characteristic classes, instead of one by one, establishes a crucial method for scientific investigation. This method was later applied in the physical domain by Gottfried Leibniz and Carl Gauss, and laid the basis for Georg Cantor's later development of transfinite numbers.

Greek scientists prior to Eratosthenes had investigated prime numbers, and Euclid (ca. 300 B.C.) recorded that knowledge in the *Elements*. Euclid showed that all numbers are either prime or composite, and that any composite number is divisible by some combination of prime numbers.

You can prove this for yourself, in the following way: Any composite number can, by definition, be divided by some other number, and that other number is either another composite number or a prime number. If it is a prime number, we need go no further. If it is a composite number, then that new composite number can be divided by another number, which is either a prime number or a composite number, and so on. By this method, you will eventually get to a prime number divisor.

For example, 30 is a composite number, and can be divided into 2, a prime number, and 15, a composite number. In turn, 15, can be divided into 3, a prime number, and 5, also a prime number. So, the composite number 30 is made up of, and can be divided by, prime numbers 2, 3, and 5.

### Constructing the Integers

Euclid also proved that the number of prime numbers was infinite. Gauss was the first to prove (*Disquisitiones Arithmeticae*, Article 16) that a composite number can be decomposed into only one combination of prime numbers. In the above examples, no combination of prime numbers other than  $2 \times 2 \times 3$  will equal 12. Likewise for 504, or any other composite number.

Hence, it is shown that prime numbers are those from which all other numbers are composed. The primes are primary. The word the ancient Greeks used for “prime,” was the same word they used for “first” or “foremost.”

This raises the question: What hap-

pens when you try to construct all integers from the primes alone? First, you'd make all the integers composed only of 2, such as 4, 8, 16, . . . . Then you'd make all the integers composed only of 3, and of combinations of 2 and 3, such as 6, 9, 12, . . . ., and so forth; then with 5, etc. As you can see, this process would eventually generate all the integers, but in a nonlinear way.

Compare that process with constructing the integers by addition. Addition generates all the integers sequentially, by adding 1, but does not distinguish between prime numbers and composite numbers.

The unit 1 is indivisible, with respect to addition. With respect to division, the prime numbers are indivisible. Both processes will compose all the integers, but that result coincides only in the infinite. In the finite, they never coincide.

This anomaly is a reflection of the truth that there exists a higher hypothesis which underlies the foundations of integers—a hypothesis which is undiscoverable if limited to the domain of simple linear addition. By reflecting on this anomaly, we begin, as Socrates says, “to see the nature of number in our minds only” (from Plato's *Republic*). Our minds ascend, as Socrates indicates, to contemplate the nature of true Being.

We ask, “If the domain of primes is that from which the integers are made, what is the nature of the domain from which the primes are made?”

—Bruce Director