TRANSLATION

The Harmony of the World (*Harmonice Mundi*)

Preface to Book I: On the Reason for the Knowledge and Proof of the Regular Plane Figures Which Create Harmonic Proportions, with their Origin, Classes, Order, and Differences

(1619)

Johannes Kepler

In the work known as Harmonice Mundi, the German scientist and mathematician Johannes Kepler (1571-1630) presented to the world his crowning work, based on the method which he had defined in his first book, Mysterium Cosmographicum (The Secret of the Universe) (1596). Many know of Harmonice Mundi as the work in which Kepler announced the third of his laws of planetary motion: the ratio of the cube of the (average) radius of the planet's orbit to the square of its periodic time, is equal to a constant for all planets. This law, which applies as well to all the planets and systems of moons discovered since Kepler, does not define the true importance of this work, however. For, in Harmonice Mundi, Kepler synthesized his studies in music, geometry, epistemology, and astronomy, to create a scientific hypothesis for the astronomical domain which opened the door to critical advances in all science.

In the Preface translated here, Kepler expresses his sense of the sacred nature of scientific inquiry, both from his descriptions of how it should be carried out, and his unabashed attacks against those who demean it. Although it serves to introduce the entire work, and many of the profound inquiries about the character of human knowledge are touched on here, this Preface refers specifically to Book I, and the foundations laid here for the rest of Harmonice Mundi. Book I is the most difficult section to read, but provides the scientific language which Kepler will need throughout the rest of the work. The language is based on the process, described in Book V of Euclid's Elements, of making incommensurable line lengths "knowable," by determining a method to construct them in defined ratios to a given line; Euclid's Book X organizes such incommensurables into thirteen divisions, or species. Kepler calls the numbers which express these relationships, traditionally known as "irrationals,"—i.e., "not capable of ratio," commonly misconstrued to mean "not reasonable"—, "inexpressibles"—"not nameable"—, to emphasize their susceptibility to reason. Kepler understands the process of determining the different species of inexpressibles in Euclid's Book X, to be a necessary precondition for *constructing the five regular (Platonic) solids in Euclid's concluding Book XIII. Kepler's first great scientific insight, reported in Mysterium Cosmographicum, had been to recognize the way in which these five regular solids determine both the number of planets as he knew them, and the relative distances of their orbits from the Sun. In Harmonice Mundi, he uses that insight, plus the coherent basis for constructing the consonant (sweet-sounding) musical intervals, to explicate the full lawfulness of the structure of the Solar System, including the relative speeds of planetary motion and the elliptical nature of the planetary orbits.*

*Kepler's view of Euclid has particular relevance when read in the context of Lyndon LaRouche's Introduction to the forthcoming Book II of the Schiller Institute's "Manual on the Rudiments of Tuning and Registration" [*SEE *page 28]. Kepler comprehended the work of Euclid as a whole, guided by a purpose: the construction of the Platonic solids in the final Book XIII. There, in the proof of the uniqueness of these geometrical figures, is contained the exposition of "Euclidean space" from the standpoint of the geometrical method identified by LaRouche as Analysis Situs. Without the rigor which Euclid supplies, and which Kepler here defends, the breakthrough of Bernhard Riemann to a non-Euclidean geometry of change, would not have been possible. Transformations of scientific outlook could not be accomplished; the thinker would be trapped, either in an existing mode of thinking, or in a world in which any proposed change would be nothing more than mere irrational assertion.*

By his passionate appreciation of Euclid's geometrical system, Kepler was enabled to take the step to his conception of well-tempered polyphony as the language of science. As he wrote at the end of the final Book V of Harmonice Mundi:

> *Accordingly, the movements of the heavens are nothing except a certain everlasting polyphony (intelligible, not audible) with dissonant tunings, like certain syncopations or cadences (wherewith men imitate these natural dissonances), which tend towards fixed and prescribed clauses—the single clauses having six terms (like voices)—and which marks out and distinguishes the immensity of time with those notes. Hence, it is no longer a surprise that man, the aper of his Creator, should finally have discovered the art of singing polyphonically, which was unknown to the ancients, namely, in order that he might play the everlastingness of all created time in some short part of an hour by means of an artistic concord of many voices and that he might to some extent taste the satisfaction of God the Workman with His own works, in that very sweet sense of delight elicited from this music which imitates God.*

Since the causes of harmonic proportions have to be sought by us in the several divisions of a circle into equal parts, which are made geometrically and scientifically, that is, from the provable, regular, plane figures, I considered that it had to be made known at the outset, that the mental characteristics of geometrical things, are today, to the extent shown in published accounts, unknown for solids. Thus, among the Ancients, no one appears who showed that he himself knew these specific characteristics of geometrical things exactly, except for Euclid and his commentator Proclus. The distribution, by Pappus of Alexandria, and by the Ancients whom he followed, of the problems arising from each part of the subject of geometry into planes, solids, and lines, was close enough to the habits of mind which have to be developed. However, his treatment is short, and applied to practical matters. No mention is made of the theory; but, unless we are occupied in our whole mind with the theory of this, we will never be able to comprehend harmonic ratios.

Proclus Diadochus, who published four books on Euclid's Book I, brought theoretical philosophy into the subject of mathematics, as is known. If he had also left us his commentary on Book X of Euclid, and had not been despised, he would both have freed our geometers from stupid ignorance, and assisted me in this labor of developing the characteristics of geometrical things for solids. It is easily shown from the Preface to his book, that these distinctions of mental existences were known to him well enough, since he established that the principle of the whole essence of mathematics, is the same as those which also advance through all forms of existence, and generate everything from themselves, that is, the finite and the infinite, or the limited and the unlimited, recognizing the limit or circumscription as the form, the unbounded as the matter, of geometrical things.

The characteristics of quantities* are shape and proportion: shape of the particular, proportion of what is joined together. Shape is completed by limits: a straight line by points; a plane surface by lines; a solid is limited, circumscribed, and shaped by surfaces. And, that which has been bounded, circumscribed, and shaped, can, then, also be comprehended by the mind. The infinite, and the indeterminate, to the extent they are such, can be con-

* That which has extension. In the standard English translation of Euclid's *Elements* by Thomas Heath, the term used for the generalized theory of extension and measurement, is "magnitudes."—SB

fined by none of the knowledge which is provided by definitions, and by no restraint of proof. But the figures exist in the Archetype, before they are in that which is produced. They are in the divine Mind, before they are in created things, with a different mode of the subject, but nonetheless with the same form of its essence.

For quantities, therefore, shape, a certain mental essence, or intellection, is made their essential characteristic. This is much clearer with proportions. Since a shape is completed by more than one bounding limit, a shape is made so that it uses proportion on account of this plurality. But what would proportion be, without an act of the mind?—it cannot be known at all. And so, therefore, for this reason, whoever ascribes limits to quantities as their essential principle, regards the quantities which have been shaped, as having an intellectual essence. But there is no need for argument: Proclus' book should be read in its entirety; it will be clear enough that he did know the intellectual characteristics of geometrical things in a provable way. However, although this is affirmed, he does not put it this way, separately, on its own, in the open, conspicuous, so that he could also admonish his own carelessness. His language flows like a flooding river, layered throughout with an abundance of the more abstruse propositions of Platonic philosophy, among which the above is the argument of this extraordinary book.

But, thus far, our generation has not been permitted to penetrate such hidden matters. Proclus' book has been read by Petrus Ramus [Pierre Ramée (1515-1572)], but, in what concerns the core of the philosophy, it has been scorned and thrown aside, along with Book X of Euclid. And, whoever wrote commentaries on Euclid, if they wrote in his defense, has been ridiculed, and ordered to remain silent. The aroused wrath of the embittered censor is turned against Euclid as against a criminal. Euclid's Book X, which, when read and understood, can unfold the secrets of philosophy, has been condemned by savage sentence not to be read. I ask you to read Ramus' words; nothing more shameful was ever written by him. In the *Study of Mathematics,* Book 21, he says:

> The matter developed in Book X has been handed down in such a way that I have never found such obscurity in human letters or arts. I say obscurity not in relation to human understanding—Euclid anticipates that (this should be clear to the illiterate and uneducated, who only look at what is right in front of their eyes)—but in relation to investigating and searching out what the purpose and proposed use of the work might be, what the classes, types, and categories of subjects might be. I have never read anything more confused and involuted. Does it not seem that the Pythagorean superstition has been drawn into this book as if into a pit . . . .

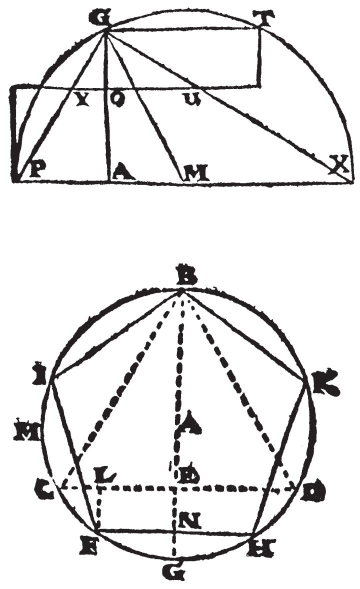
By Hercules, Ramus, had you not believed that this book would be easy to understand, you never would have slandered it so much as obscure. You need more work. You need quiet. You need forethought. And, above all, you need attentiveness of mind. Then, might you understand the intent of the writer; from thence, the good sort of mind will be lifted up to the point where, resolving to live at last in the light of truth, it is inspired, exulting with incredible joy, and perceives the whole universe and all its different parts in the most exact way, as if from a mountaintop. But, for you, who act in this place as the advocate of ignorance, and of the common man seeking advantage from everything, whether divine or human—I say, for you, these matters may be "unnatural sophistries," for you "Euclid will have abused the quicker thinker immoderately," for you "this subtlety has no place in geometry." Let it be your lot to slander what you do not understand. But, for me, hunting for the causes of things, no other path will lead to them apart from that which is in Euclid's Book X.

Lazarus Schoener followed Ramus in his geometry; he confessed that he was not able to see any use in the world for the five regular solids. Then, he read the book which I entitled "The Secret of the Universe," in which I prove that the number and orbits of the planets were chosen from the five regular solids. Now, look at the damage that Professor Ramus did to his student, Schoener. First, when Ramus read Aristotle, who had refuted the Pythagorean philosophy concerning the way in which the properties of the elements derive from the five solids, he immediately conceived a contempt for the whole of the Pythagorean philosophy. And then, since he knew that Proclus had been part of the Pythagorean cult, he did not believe that which Proclus affirmed, which was most true, that the ultimate purpose of Euclid's *Elements,* to which all the propositions of all the books taken together are related (with the exception of those on perfect numbers), is the five regular solids. From this, there rose up in Ramus the most shameless belief, that the five solids must be removed from the conclusion of Euclid's *Elements.*

When the conclusion of the *Elements* was chopped off, only a formless heap of propositions was left of Euclid, like the rubble of a levelled building, against which, as if against some ghost, Ramus inveighs in all the twenty-eight books of his *Study of Mathematics,* speaking with great harshness and rashness, most unworthy of such a man. Schoener followed this belief of Ramus, and thus himself believed that there is no use for the regular solids.

Not only this, but he followed Ramus' judgment, and despised and scorned Proclus, from whom he could have learnt the use of the five solids, both in Euclid's *Elements,* and in the making of the universe. However, the student was much happier than the professor, since he joyfully accepted the use of the solids in the making of the universe, as disclosed by me, which Ramus had refused to impress upon him from Proclus.

But, then, so what if the Pythagoreans attributed these figures to the elements [earth, air, fire, water, and the heavenly "quintessence"—SB], and not, as I do, to the spheres of the universe? Had Ramus exerted himself to remove this error of theirs about the real subject of the figures, as I did, he would not have come up with one tyrannical word against this whole philosophy. Or, what if the Pythagoreans actually taught the same thing I do, while hiding their meaning in a protective cloak of words? Is not the Copernican form of the universe in Aristotle, and falsely refuted by him, but under other names, as when they called the sun, "fire," and the moon, "Antichthone"? For if the ordering of the orbits was the same for the Pythagoreans as it was for Copernicus; if the five solids and the necessity for their five-fold number was known; and, if they all uniformly taught that the five solids are the Archetypes of the parts of the universe: how little more would it take for us to believe that their opinion was read by Aristotle in its secret form, but had been, as it were, refuted by the literal meaning of the words?



Diagrams from Harmonice Mundi, Book I

Thus, where Aristotle read [the element—SB] "earth," as that to which they assigned the cube, those men might perhaps have understood it as Saturn, whose orbit is separated from Jupiter by the interposition of a cube. And, the common sort of people attribute rest to earth; thus, Saturn has been allotted the slowest motion, the closest to rest, for which reason the planet was given the name "rest" by the Hebrews. In the same way, Aristotle read that the octahedron was assigned to "air," when they might perhaps have understood by this, Mercury, whose orbit is contained by an octahedron; and, Mercury is no less fast (it is certainly the fastest of all the planets) than the air is mobile. Perhaps Mars was implied by the word for "fire," since the name of this planet otherwise is "pyrois," which is derived from the Greek word for fire, and perhaps the tetrahedron was assigned to this because its orbit is enclosed by this figure. And, under the cover of "water," to which the icosahedron is attributed, the star of Venus could be hidden (its orbit is contained by an icosahedron), because fluids are subject to Venus, and she herself is said to have been born from the ocean spray; hence, the name, Aphrodite. And lastly, the sound of the word "world" could have meant Earth, and the dodecahedron could have been allotted to the "world," because the orbit of the Earth, which is bounded by this figure, is divided in twelve longitudinal parts [the divisions ("Houses") of the astronomical Zodiac—SB], as this figure is bounded by twelve planes for its whole extent. If this is accepted, then, in the mysteries of the Pythagoreans, the five solids would not have been distributed among the elements, as Aristotle believed, but among the planets. This, if you will, is strong confirmation of what Proclus handed down, among other things, as the purpose of geometry, and what he taught, about the way in which heaven would have taken upon itself figures consistent with its defined parts.

And this is not yet the end of the injury which Ramus does us. Behold Snell, the most skillful of today's geometers, giving open support to Ramus. In the preface to "The Problems of Ludolph of Coellen [Cologne]," he says at the start that "the very division of the inexpressibles into thirteen different types is of no profitable use."* I grant that, if he does not recognize any uses, except for everyday life, and if none of the investigations of physics would be useful for life. But why does he not follow Proclus, a man whom he admits recognizes some greater good in geometry than the arts which are necessary to life? But, then, the use of Book X would obviously have been clear, in evaluating the types of figures. Snell brings forward authors of works

* These "inexpressibles" are incommensurable magnitudes, which are constructable, but cannot be expressed as ratios of whole numbers; therefore, no precise numerical value, or "name," can be given them. In Book X of the *Elements,* Euclid divides the inexpressibles known to Greek mathematics into thirteen distinct species.—SB

on geometry who do not use Book X of Euclid. All these, of course, deal either with problems of lines or solids, and with figures, or such magnitudes as do not have their purpose within themselves, but obviously tend toward other uses, and would not be investigated without those other purposes. But the regular figures are investigated on their own account as Archetypes; they have their perfection in themselves; and they are counted among the problems of planes, for a solid is enclosed by plane surfaces, and, likewise, the most important subject matter of Book X pertains to planes. Therefore, why would anything else be brought forward? Or, why are the goods which Codrus does not buy to stuff his belly, but Cleopatra does buy to decorate her ears, thought to be so worthless in value? "Is torment so fixed in our way of thinking?" Certainly, for those who vex the inexpressibles with numbers, that is, by expressing them [i.e., numerically—SB]. But, I do not use numbers to discuss these types, nor algebra, but the reasoning of the mind; since, it is not necessary for me to use these to add up a shopkeeper's accounts, but to develop the causes of things.

It is thought that these subtleties ought to be separated out from the *Elements,* and stuffed away in the archives. That is what Ramus' trustworthy student is getting at, and he is not idly making a point. Ramus destroyed the form of the Euclidean construction, he tore down the high-point of the work, the five solids: after these had been removed, the whole structure was destroyed, cracked walls are left standing, jutting arches lie in ruins: then Snell took away the cement as well, for there is no use for his solid house if not cemented together under the five figures. What more productive discovery by a student could there be, than his affirmation that by good fortune he had gained an understanding of Euclid from Ramus. They think the *Elements* are so called, in that there is found in Euclid an abundant variety of propositions, problems, and theorems, for every type of quantity and of the arts that are concerned with these; whereas the book may have been named *Elements* from its form, in which a subsequent proposition is always supported by the one that precedes it, right through to the last proposition of the last book (and partly, through Book IX), which cannot stand without everything that preceded it. Out of the architect, they make a forest ranger or timber merchant, by thinking that Euclid obviously wrote his book so that it might be the storehouse for all others, and he alone should have no dwelling of his own. But, this is more than enough of these matters at this point; we must now return to the main line of our discussion.

When I understood that the true and real characteristics of geometrical things, from which I had to derive the causes of harmonic proportions, are generally not known in any depth; that Euclid, who carefully handed them on, is dismissed and suppressed by the mockery of Ramus, and because of the confused babbling of the stupid, is heard by nobody, or else tells the secrets of philosophy to the deaf; and that Proclus, who could have opened up the mind of Euclid, uncovered hidden things, and made what is difficult to understand, easy again, is an object of derision, and did not continue his commentary up to Book X; I saw that all this had to be done by me. So, to begin with, I wanted to transcribe from Euclid Book X, those things which would contribute in a special way to my present undertaking; I wanted to shed light on the series of things in that book, by interposing certain definite divisions of the subject matter; I wanted to show the reasons why some parts of the divisions have been omitted by Euclid; and then, lastly, there must be a discussion of the figures themselves.

I have been content to simply refer to the propositions in the cases which were proven most clearly by Euclid, but there are many questions, which have been proven by Euclid in a different way, which now, on account of the purpose that has been given me, that is, the comparison of the figures which can be known with those which cannot, must be reworked or joined together again where they have been separated, or the order changed. I have combined the series of definitions, propositions, and theorems, in numerical order, as I did in the *Dioptics,* for the ease of reference. I have not been accurate with regard to lemmas, nor over-anxious about names, as I am more concerned with the constructions themselves. Clearly, this is not geometry in philosophical terms yet, but in this part I do discuss the philosophy of geometry. I would like to have been able to deal in a still more popular way with the questions of geometry, provided the treatment were clearer and more palpable. But, I hope that readers equal to both will take my work in good part, in that I both teach geometry popularly, and was not able to overcome by my efforts the obscurity of the subject matter. Finally, I give this advice to any readers who might be completely unfamiliar with mathematical questions: They should pass over the narrative, and read only the propositions from Number 30 to the end, and taking those propositions on trust, they should proceed to the other books, especially the last. If such readers should be terrified by the geometrical argument, they might deprive themselves of the most joyful fruit of the harmonic investigation. Now, let us go to work, with God.

—translated by Sylvia Brewda and Christopher White,
assisted by Molly Kronberg