Click here for Full Issue of Fidelio Volume 5, Number 4, Winter 1996

The Invention of Perspective

Thoughts on how the science of perspective came into being

by Karel Vereycken

The invention of perspective was a giant step for mankind, through which humanity greatly increased its mastery over nature. Lost for centuries and rediscovered only in the Renaissance, when an explosion of genius gave it accelerated development, this science was the result of protracted effort, and involved a great many superseding hypotheses. Here, we review the outlines of this historic debate.

Before considering the technical notions pertaining to constructing images in perspective, let us first look at some elementary issues. First: How is it that that marvel, the human eye, allows that miracle known as sight, to occur? What feat is it, that enables us to grasp the complexity of the world around us?

That answered, we shall examine with a suitable degree of suspicion, different representational systems, before discarding them.

Rather like learning to swim, there is no way around an initial feeling of unease. To avoid going under, you must not be too heavy, nor thrash around too much—because here you are your own lifeguard. Eschewing the formal logical presentation of yet another theory, we prefer to set up paradoxes, which are designed to afford points of reference to the novice, while spurring on those who have some grasp of the subject.

Rethinking Vision

Despite a bold effort by some neurophysiologists over the last decade, the phenomenon of sight, as engraved in collective consciousness, is generally thought of as being roughly similar to a *camera obscura,* or dark room [SEE Figure 1]. The brain is imagined to be a sort of huge computer hooked up to a hyper-sensitive camera: each time there arises a stimulus to the cones (color reactive) and rods (depth reactive) of the retina (from the Latin word for network, *rete*), there is a corresponding stimulation of a point on the visual projection area of the cortex. Known as the *internal screen theory,* according to which the brain would be a kind of movie theatre, it contends that, *first,* outside images are projected by our organs of sight onto an internal screen, and only *afterwards,* are they interpreted by our consciousness. Such a theory reflects, predictably, the philosophical dualism of Aristotle, Descartes, and Newton: man, the "mind-subject," objectively interprets the "matter-object," or world. Were this mechanistic view shown to be correct, we should shortly be able to put together machines better able to see than any man, and creative computers, better able to think than any scientist.

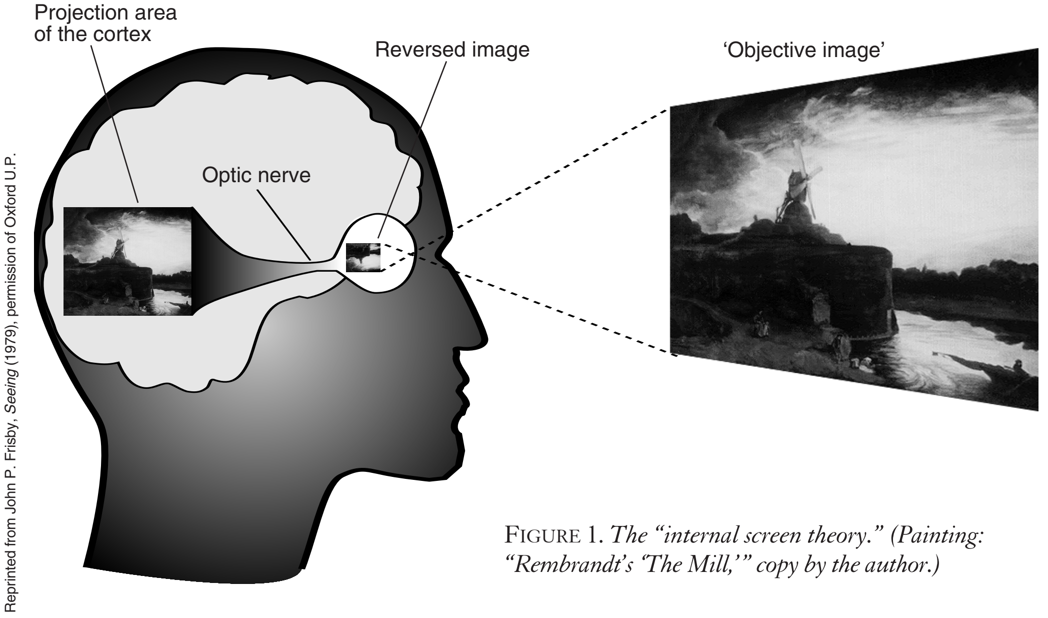
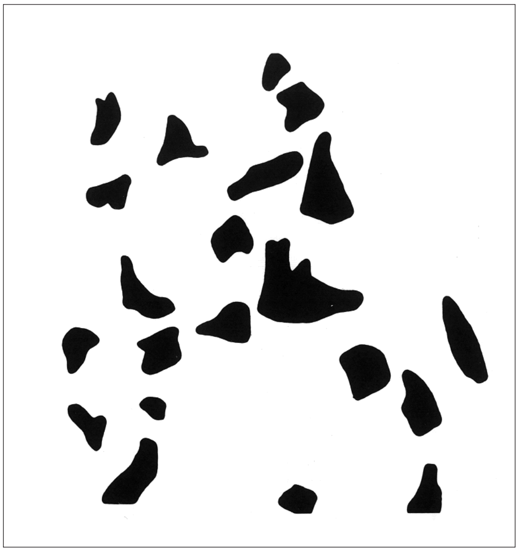


FIGURE 1. *The "internal screen theory." (Painting: "Rembrandt's 'The Mill,'" copy by the author.)*

Although it is of some use as a heuristic model, the "camera obscura" theory is, nevertheless, a gross simplification; a detour past images sometimes called *optical illusions* will uncover its fallacious side. These optical illusions are intended to shatter our belief in the objective nature of "photographic perceptions," and to raise the veil shading the true character of the function of sight. How curious, to see that you may not be seeing what you see, if you see what I wish you to see . . . .

Study of the two images in Figure 2 shows that the act of seeing calls for some sort of intellectual grasp of *what* is seen. Once we have established what the image means, we cannot put that meaning out of mind. As soon as the Dalmatian's spots and the horseman's parts have become "blindingly" obvious, these images never again appear to us as a collection of black spots lacking rhyme or reason. The puzzle having once been solved, the image of the puzzle as a whole reveals itself to the mind's eye in each and every one of its pieces. In other words, seeing is an act of man's will, utterly different from the action of the camera, which does not see, but merely records. Sight, it turns out, is, in fact, a complex function, having to do



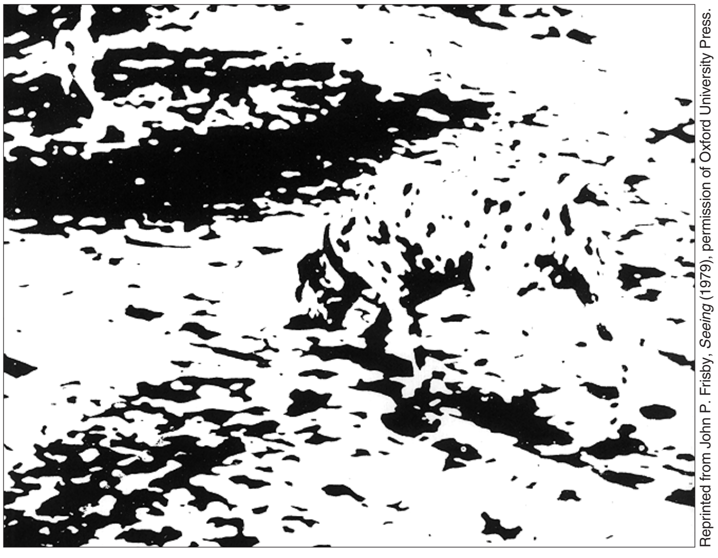
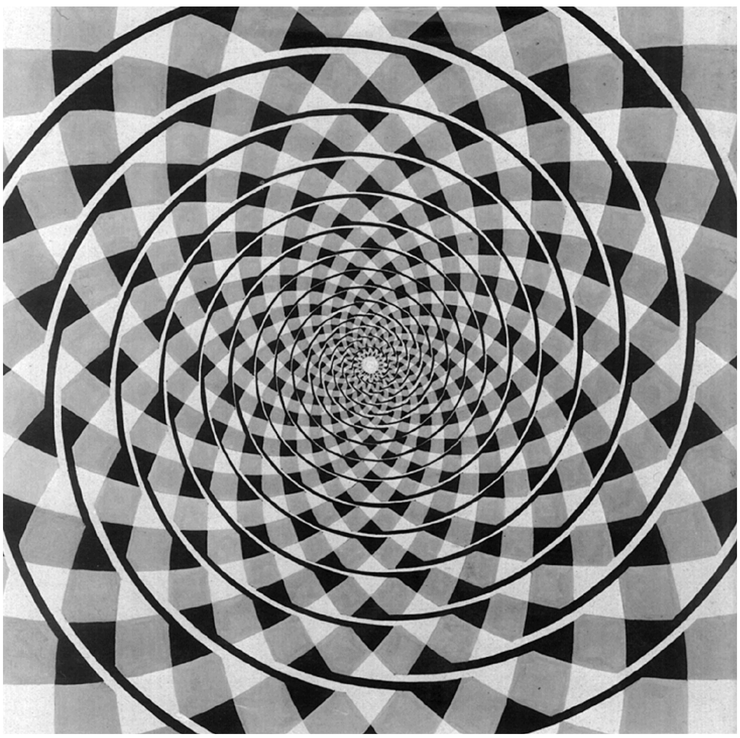


FIGURE 2. *Optical illusions. The eye seeks to make sense of these spots straightaway. Once the meaning has been found, it cannot be "unlearned."*

with how one finds things out, how one conquers new areas of thought; it is an act of cognition. It is worth noting here, that in the human embryo, the eyes and the brain develop out of one single original unit.

We have said that, unlike the camera, the organs of sight are not "objective." This point is well made by the famous case of Dr. P., as reported by the celebrated neurologist Oliver Sacks in his account, *The Man Who Mistook His Wife for a Hat.* A patient of Sacks, Dr. P., suffered from a disorder affecting the brain's visual zones. When, for instance, Dr. Sacks showed Dr. P. a glove, the latter identified the glove as a continuous surface with five outgrowths which seemed to him to be a kind of receptacle. Thus, the patient saw the details (the Many), but not the image in its entirety (the One). Dr. Sacks concludes: "Visually, he was lost in a world of inert abstractions. Clearly, he had quite lost contact with the real visual world, in the same way that he no longer possessed, so to speak, a *visual self.* Doctor P. operated as though he had become a machine. Not only was he as indifferent as a computer might be towards the visual world, but, more striking still, he broke the world down into parts as a computer does . . . . He was clearly unable to come to any cognitive judgment . . . ." [translated from the French edition—KV]

FIGURE 3. *"Fraser's Spiral."*



Reprinted from John P. Frisby, *Seeing* (1979), permission of Oxford University Press.

Dr. Sacks further reported that Dr. P., an amateur painter, had moved away from figurative to abstract painting precisely *because* of his pathology.

From the preceding, it is apparent, that were the visual function nothing but a rush of details travelling through our field of perception, man would never have survived as man, but would have rather vegetated, in the manner of someone hallucinating, the prey to images wandering in free association through his mind.

But what is it, that we do see? Is it mind or matter, rest or motion; or, is it something else?

In the Fifth Book of *The Republic,* Plato's *Myth of the Cave* raises the issue in this way: Prisoners held in a cave, in chains, are made to bend their gaze to a wall upon which shadows are cast. Are these shades the *All* of reality? To the prisoners, to whom the shades are objects in themselves, it is so. If one were to get free of the cave and come out into the light of day, dazed and blinded, his first and only impulse would be to flee back to *familiar reality,* back to the shadow-objects cast upon the wall. But, should he once become accustomed to the light, the idea may occur to him that *behind* the shades, there is a reality, revealed and made intelligible, in part, by enlightened interpretation of simple perception.

The prisoners' chains stand for the limitations of our senses, the which lead us to confuse the *perception* of an object with its *reality.* Subjective beings as we are, we have no access to the objective reality of a thing; it is only by the force of reason, that we are taken beyond our limits, on toward the *truth* of a thing, that is, its *idea.*

In "Fraser's spiral," to our surprise, we find that, although it is spiral action which dominates what we see in this image, we are dealing, in point of fact, with concentric circles! [SEE Figure 3] The illusion of the spiral is so powerful, that even if you trace along the circles with your finger, the illusion may yet pull you into its orbit—there, where a computer would "see" nothing but concentric circles. A computer could never "perceive" the idea of spiral motion, which motion is nonetheless quite real.

So far, we have shown that man sees much more than "forms" dotted about the landscape. It would appear that sight obeys the principle of least action—(how the least possible effort may be applied to produce the greatest possible quantity of work), a principle that occurs everywhere in the spatial ordering of organic growth, and in the geometrical organization of technologies applied by man. We are led by the higher functions of mind directly to the essential, i.e., to see Transformation, Action, and even *potential* Action: in the matter of vision, essence precedes existence. The mind, conforming in this with the laws of the universe, is directed entirely toward grasping the primacy of processes of transformation, whether they be actual or potential; processes, where mind and matter are as one. Witness, the stairs in the drawing "Ascending and Descending" by M.C. Escher [SEE Figure 4]. Men are clearly to be seen going up and down steps; the fact that they always come back to the same starting point does not disturb us overmuch! A trick with the perspective makes the building's fake geometry seem perfectly plausible, because that geometry breathes action, which takes over the entire image. Thus, the idea of action is so overpowering, that it can even lead us into error.

Why Perspective?

Once it has been understood that to *see* means to make *intelligible,* it must needs follow that to *depict* a thing, means to *make others see;* in other words, to make it intelligible to one's fellow men. Drawing is first and foremost a *language,* or, if you will, several languages. Indeed, an architect will not use the same terms with his builders, as with those who are to live in the house.

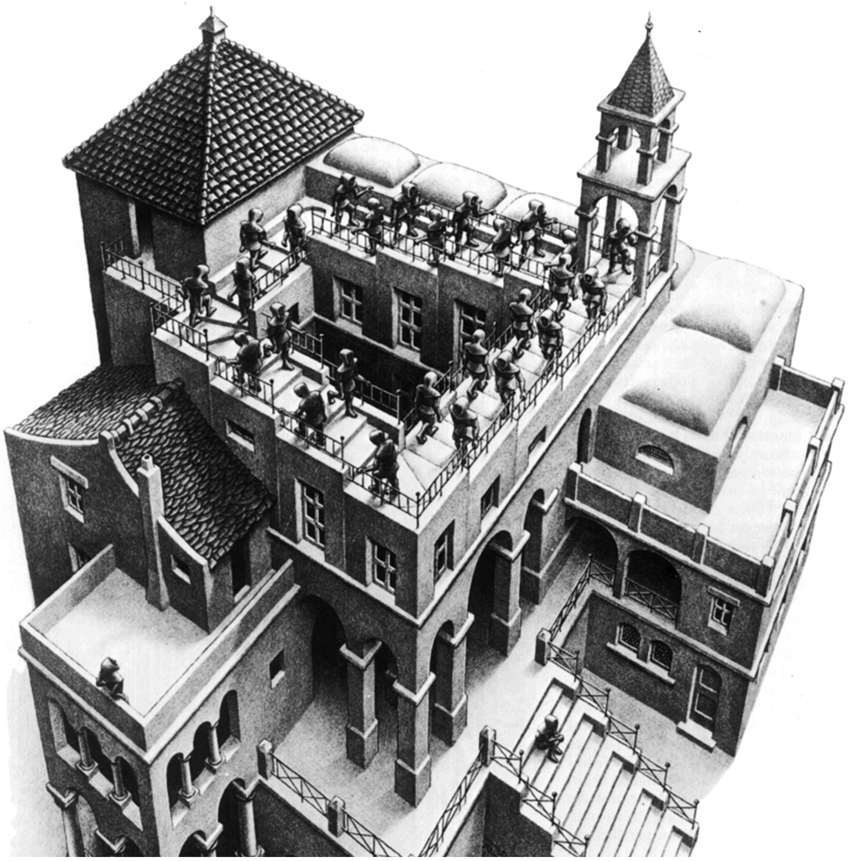


FIGURE 4. *M.C. Escher, "Ascending and Descending," (detail).*

The contractor and the builders will be given detailed blueprints with all they need to know to put up the house: its various dimensions, each of the materials to be used and, so on. Whereas, those who will live there, will be shown a glowing perspective, so that they may admire the depth of the living room, or the cunning spiral staircase. With his builders, the architect refers to the *object;* with his clients, to the *idea*.

In order that we may communicate those elements needed to build a three-dimensional object, recourse is had to projective geometry, which involves both isometry and the notion of scale. Projective geometry emerged from a process begun in Paleolithic times, when man realized he could project onto a cave wall, in outline, that best of all tools: his own hand [SEE Figure 5].

Over thousands of years, countless experiments led to the breakthroughs made by the engineer-architects of the Ecole Polytechnique, Gaspard Monge and

FIGURE 5. *Silhouette of human hand, Grotte Chauvet, France, c.35,000 B.C.*

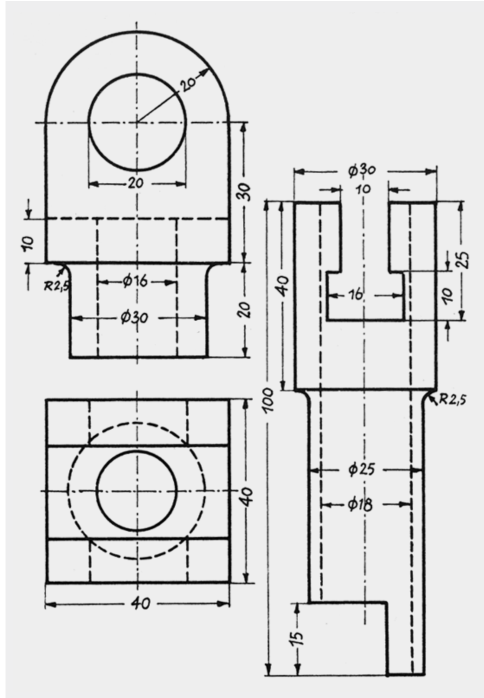
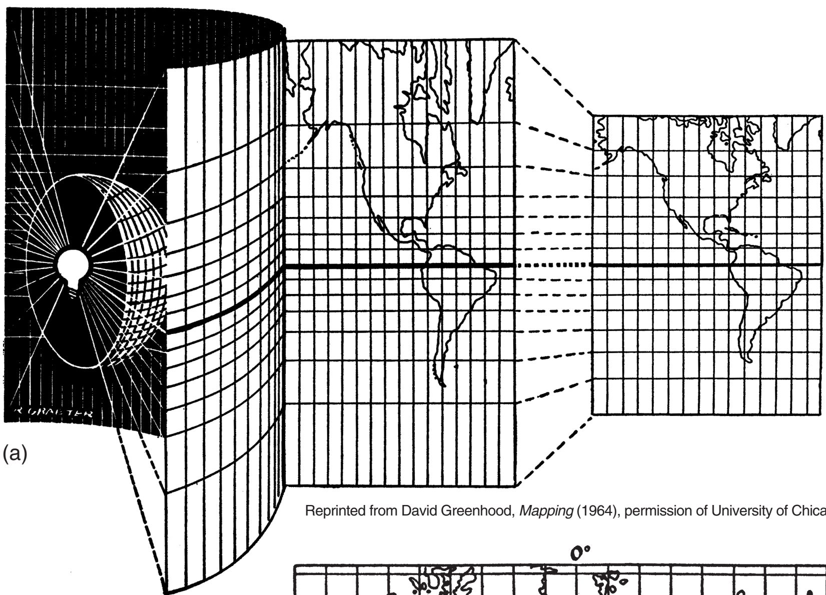


FIGURE 6. *Without projective geometry, there would have been no industrial revolution. Once it became feasible to represent an object on a plane surface, and to specify its material constraints, productivity in the economy surged upwards.*

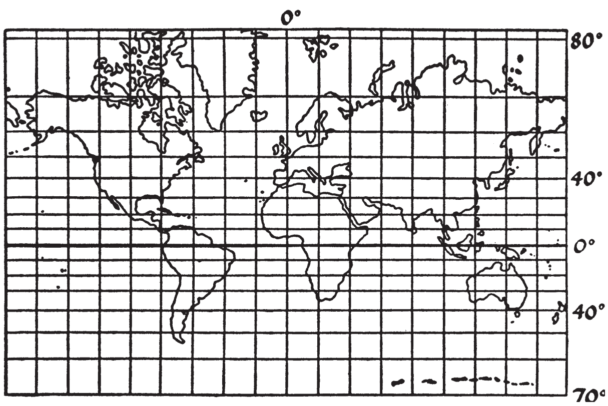
Jean-Victor Poncelet; experiments carried out by figures such as the cathedral builder Villard de Honnecourt; then in the Renaissance by Paolo Uccello, Francesco di Giorgio, Leonardo da Vinci, and Piero della Francesca, among others; and down through Gérard Desargues and Blaise Pascal in more modern times.

Owing to the development of projective geometry, there was no longer any need for wooden models to build artillery pieces and machine-tools; thenceforth, they were built straight from drawings. The new intellectual instrument made it possible to ensure that a given construction could be built over and over to the same identical specifications; by opening the way to mass production, projective geometry took mankind from the age of craftsmanship, to the age of industry. *It was not the Renaissance that created "perspective," but perspective as a science that gave rebirth to civilization.* Its consequences were so far-reaching, that in France, until the Revolution, the new geometry was jealously guarded as a military secret; it was to become the key-



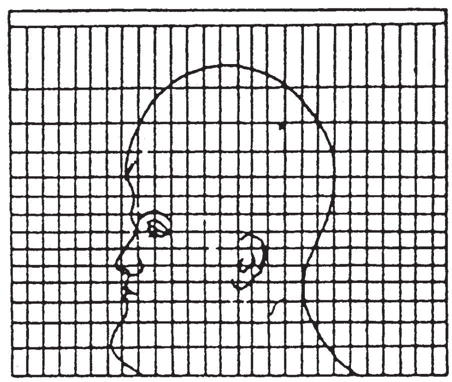
(a)

Reprinted from David Greenhood, *Mapping* (1964), permission of University of Chicago Press.



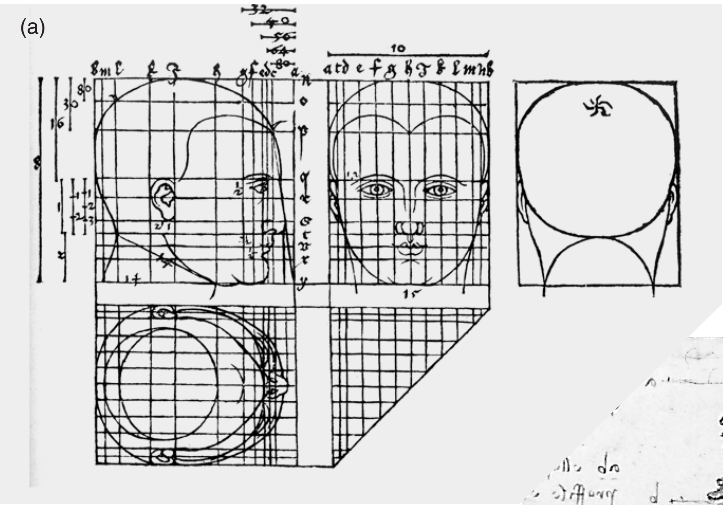
(b)

Reprinted from David Greenhood, *Mapping* (1964), permission of University of Chicago Press.



(c)

FIGURE 7. *(a) Mercator projection. (b) We are used to this map, but it is distorted. (c) Human head drawn according to the Mercator projection shows how the proportions are deformed.*



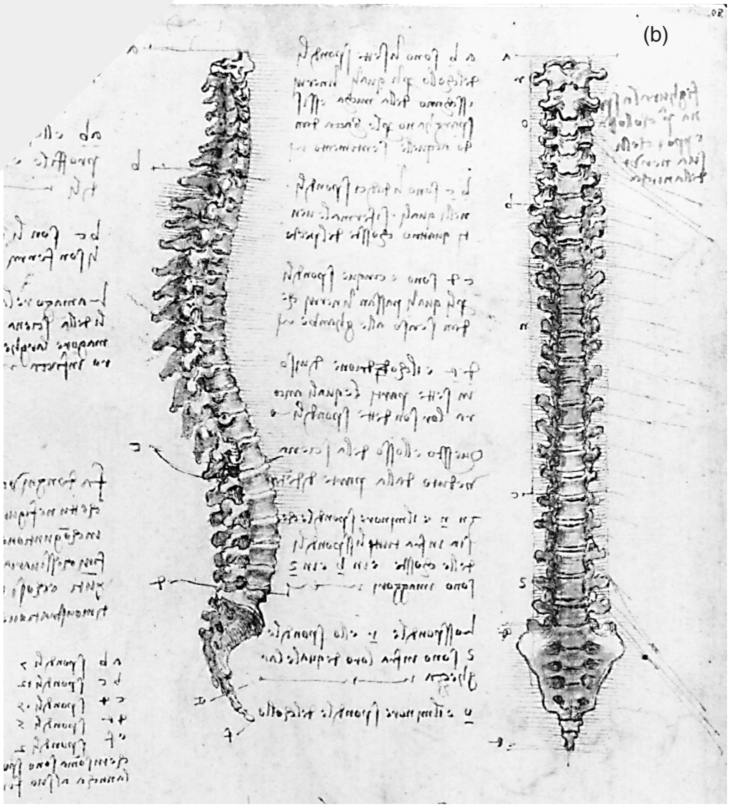


FIGURE 8. *(a) An example of Dürer's work on the measurements of the human body (1512). (b) Leonardo, who had quite another approach, had realised through careful study of the skeleton and muscles that one cannot understand living processes merely by measuring their external forms. ("Two studies of the spine," Windsor, RL19007v, detail)*

stone of the curriculum of the first Ecole Polytechnique [SEE Figure 6].

Although descriptive geometry did markedly increase the power of man over nature, it has limits one cannot ignore. The first to run up against them were the cartographers. For, although when a cube is projected onto a plane surface, nothing is altered in its essential characteristics, this is not the case with a sphere. This brings up the vexed question of the squaring of the circle, the issue dealt with by Cardinal Nicolaus of Cusa, himself an expert cartographer. Cusa showed it to be *ontologically* impossible that a true circle should ever be drawn by the procedure of adding ever more sides to an inscribed polygon.

To the demand for a cartographic topology suited to navigation, Gerhard Kremer, generally known as "Mercator" (1512-1594), responded with a projection. When the surface of a sphere is projected onto an imaginary cylinder, which is then unrolled, a map may be drawn which preserves the angular relations [SEE Figure 7]. This latter property is essential to navigation. Of course, in Mercator's projection, the continents' true relative proportions are quite distorted, increasingly so toward the Earth's poles. The latter, which were points upon the sphere, become lines on the plane surface. Thus, the sphere reveals that there is a peculiar quality to three-dimensional space, a quality which cannot be reduced to a plane surface, nor projected from a linear standpoint.

Further limits to descriptive geometry appear once one turns to examine living processes. It is most instructive, in this respect, to compare the anatomical studies by Dürer, to those of Leonardo da Vinci [SEE Figure 8].

In the wake of the excitement aroused by the studies of Piero della Francesca and of Uccello, Dürer decided to apply himself most zealously to measuring the *outside* forms of the human body. Without meaning in any way to belittle Dürer's important contribution, it must be said nonetheless, that he fell into a trap. Never did he really come to understand the *dynamic* of the human "machine," but rather wandered off down the path of a kind of geometric numerology.

Not at all like Dürer in his approach,

Leonardo looked instead at the interaction between the spinal column, which his research into anatomy had shown him to be the foundation of all movement, and the muscular apparatus. In this manner, he arrived at an understanding of what appears to us as *grace,* visible through form; as the necessary expression of work done by the body at a precise moment.

The question now posed, is whether there be a means to reach beyond the limits of projective geometry, such that there should be made intelligible all that pertains to the *idea* of creation, rather than to its *results.* But, in order for that to occur, the doctrine known as "*mimesis,*" whereby Aristotle affirms that the purpose of art is to but *imitate* nature, must be put entirely aside.

To the Aristotelean, from the fact that an idea, that movement, that transformation or the infinite, do not belong to the material world, one may deduce that suchlike notions cannot be represented, unless it be by symbols. Negating, as they do, creation as a universal law—negating, therefore, the harmonic interaction between mind and matter—, they seek arbitrarily to bind an idea to some object. France, for example, shall be represented by the tricolors Red, White, and Blue. To this school of thought, the representation of an idea is not intelligible as such, but rather, it is something to do with convention, accessible only to the initiate. To the non-initiate, it shall forever remain a mystery. How very distant is this school from the notion that creativity shall be made intelligible to the many!

That, to the Aristoteleans, beauty must be founded on two elements, i.e. *magnitude* and *order,* shows up as yet another flaw in their dualistic system. In the Seventh Book of his *Poetics,* Aristotle has written that

> a beautiful object, whether it be a living organism or any whole composed of parts, must not only have an orderly arrangement of parts, but must also be of a certain magnitude; for beauty depends on magnitude and order. Hence a very small organism cannot be beautiful; for the view of it is confused, the object being seen in an almost imperceptible moment of time. Nor again, can one of vast size be beautiful; for as the eye cannot take it all in at once, the unity and sense of the whole is lost for the spectator; as for instance if there were one a thousand miles long. (1450b)

That the Mind may be greater than the limits of sight, is something Aristotle would not even contemplate; once a thing is too large or too small to be seen, we can neither know, nor understand it. What cannot be perceived by the senses, is not, to Aristotle, part of the real universe. What's more—there being no necessary relation between objects and the space they occupy—, there is nothing left, but to be "practical," and uphold "order" by assigning to each and every object its appropriate pigeonhole.

At the opposite pole to this school of thought, lies that of Plato. Beauty, to Plato, has to do with *harmony* and *proportion*; the latter being the expression by which the underlying harmony shall be made known, and each element of Creation, an instrument by which the harmonic web of the whole shall be made known. Thus is the whole found in the part, the One in the Many.

Once we place our trust in such a preexisting—although not unchanging—harmonic Unity, there may be introduced the notion of a *horizon,* a singularity in the nature of a metaphor (in Greek, "*metaphor*" means "to carry beyond"); which notion unleashed a revolution in the science of perspective. Although this frontier does seem to appear at the seaside, it, nevertheless, has no material being as such. It can neither be measured algebraically, nor can its distance from us be calculated. The line drawn to express the horizon, is *neither object nor symbol.*

Truly a transfinite, the horizon—(while pertaining to the world of finite things, it is yet a lever to the infinite)—remains naught but a line you may easily trace; for example, the line you trace when sketching a room in your home. The horizon enfolds within it an infinite number of vanishing points, upon each one of which coincide an infinity of harmonic relations; the latter's proportions do not change, although their spatial projection decrease. That parallel lines do meet at infinity, well expresses the notion of perspective: that so harmonic, so unique an organization, encompassing all of a Creation itself so varied and so profuse, should yet be made intelligible.

The horizon, as the examples we shall present shall show, may be perhaps but one—*the first*—of the transfinites one may bring forth, while others are in gestation, so to speak. To the artist, a fixed system exists only to be transcended; to awaken the powers of mind, there must be irony, there must be surprise. Whereas measurement, *ergo* repetition, is the language of the geometer, that of the artist is movement, change, and that beauty which arises out of a lawful break with whatever order be already given, to reach a higher form of order. Only such a science of perspective is compatible with the laws of mind.

The Various Types of Perspective

Let us now examine various models of spatial representation. In the interest of simplicity, we have arranged them into three categories:

- Infantile and/or symbolic perspective
- Linear perspective
- Non-linear perspective

One type need not exclude another. In order that the artist be free to "tune" his work in accordance with that which he wishes to say, a painting may be built around the articulation between various types of perspective—rather like the way repetition may be a feature of a poem, without it being a method or *sine qua non* upon which the poem stands or falls.

Infantile and/or Symbolic Perspective

To a child, the existence of objects is self-evident, as he cannot identify processes in the real universe. Did he wish to represent "objects" or his feelings about them, he would set out by enumerating them, lining them up, like the child's drawing shown in Figure 9. Once the line has been filled in, he may perhaps draw a second line, thereby building what some call "*register perspective.*" In the same *ordering,* he might sketch in people, whose size will depend upon how important they are to him. Be all that as it may, we remain within a flat universe called an "aggregate space," rather than a "system space."

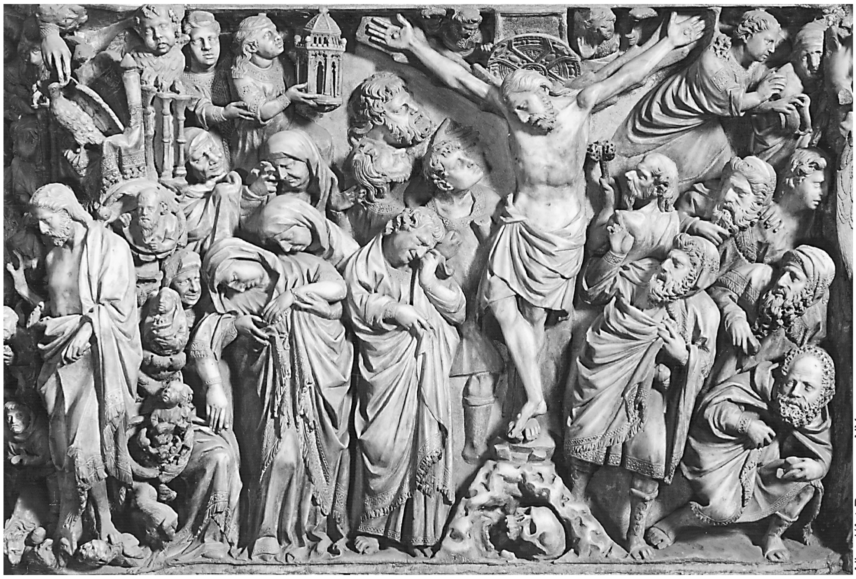
FIGURE 9.
Infantile perspective.

Pretentious as it is, modern art, too, does seem to rest upon this *non-system.*

Linear Perspective

Those who were first confronted with the problems posed by complex spatial representations, were undoubtedly the sculptors. Where a "Last Judgment" might perhaps be felt to have been adequately rendered by the low-relief ("*méplat*") technique of *bas relief,* in very complex scenes such as the Passion, the figures simply had to be brought forth from a plane surface, this truly three-dimensional technique being known as "*ronde bosse*" ("high-relief"). In the Roman and Byzantine style, a carved figure was as though caged within a plane; a revolution erupted with the Gothic style: its figures were placed within a space proper to them, often cylindrical segments of a vault, although much

FIGURE 10. *Left: "Register perspective," as seen in the "Doubting Thomas" from the Santo Domingo de Silos cloister in Burgos. Right: Nicola Pisano's "Crucifixion," for the pulpit of the Cathedral at Siena.*

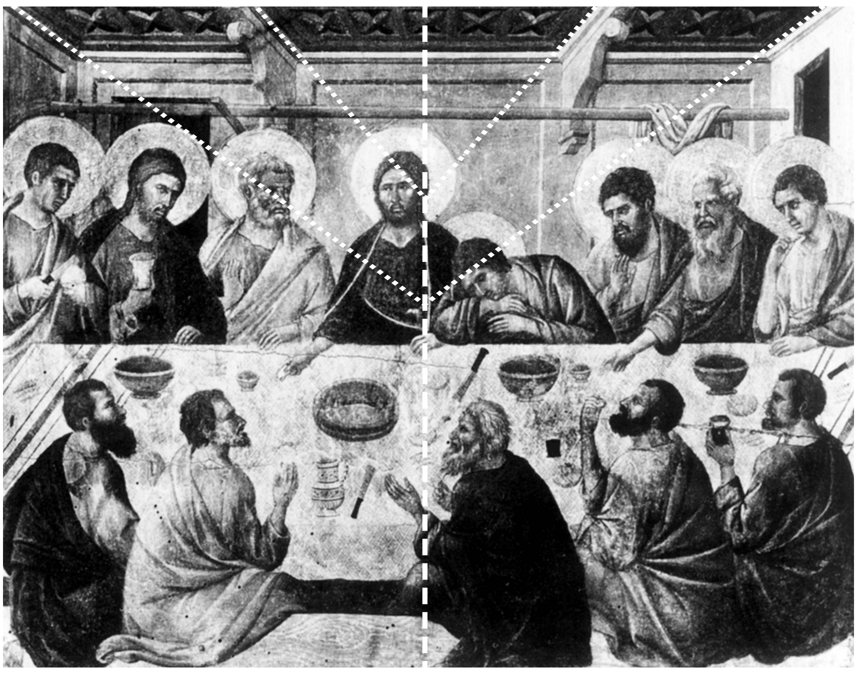


Alinari/Art Resource, NY

larger spaces were also used.

In this respect, a noteworthy comparison is that between Nicola Pisano's "Crucifixion" on the pulpit of the Cathedral at Siena (1265), and the "Doubting Thomas" at the Cloister of Santo Domingo de Silos at Burgos (c.1130) [SEE Figure 10]. Through the development of the Gothic style, three-dimensional space suddenly appears to us—(or should we perhaps say, re-appears!)—and thereby, the play of light in all its splendor. That Robert Campin (the Master of Flémalle), Jan van Eyck, and others, often depicted sculpture "*en grisaille*" ("in grays") may perhaps be their homage to the Gothic stone-cutters.

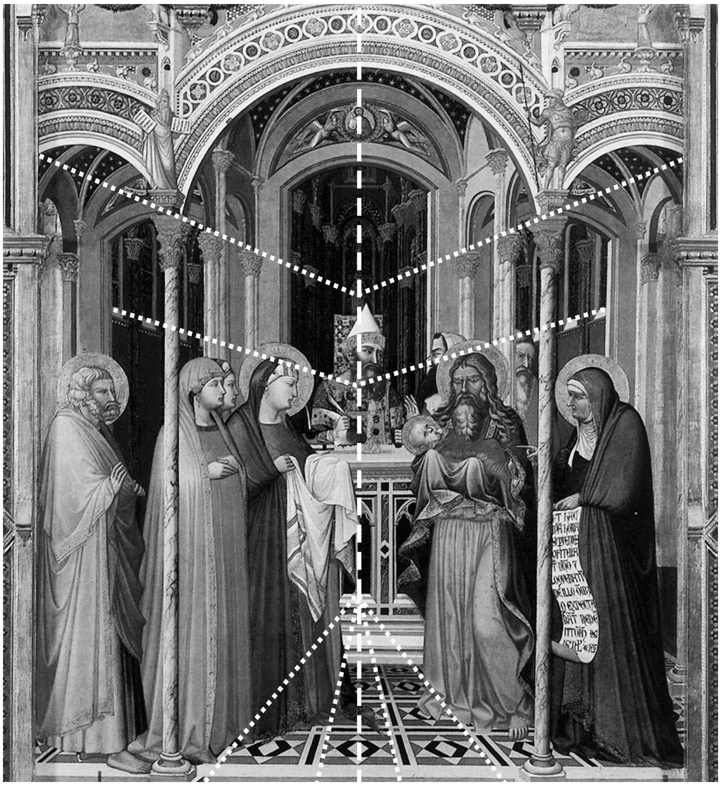
FIGURE 11.
Duccio (1255-1318), "The Last Supper," perspective in the "fishbone system."



Alinari/Art Resource, NY

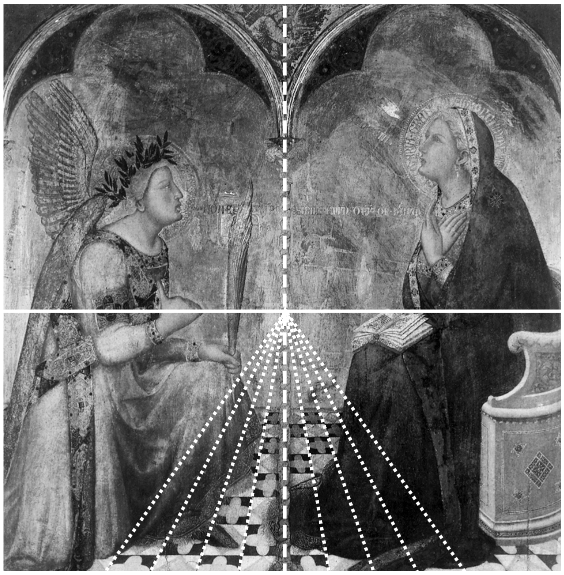
How to unify visual space, how to make it appear to be homogeneous, occupied the thoughts of those artists who first tried their hand at linear perspective. An early, Greco-Roman representation, like that one may see at Pompeii, does not rely upon a single central vanishing point, but upon a "vanishing axis," also called a "fishbone system." An example is Duccio's "Last Supper" [SEE Figure 11]. (Whether the lack of a single point at infinity arises merely from a lack of developed knowledge, or from a theological aversion to directly representing "the infinite," is not known.)

The next step was to improve upon the system, by connecting lateral vanishing lines to the central vanishing axis, at different heights, as in Ambrogio Lorenzetti's "Presentation in the Temple" (1342) shown in Figure 12. Then, however, in his "Annunciation," painted in 1344, Lorenzetti adopts one single vanishing point [Figure 12]; the question remains whether this may not be due simply to the arrangement he had decid-



Alinari/Art Resource, NY

FIGURE 12. *"The Presentation in the Temple" (left) and the "Annunciation" (right) by Ambrogio Lorenzetti, who seems to have used a hybrid system.*

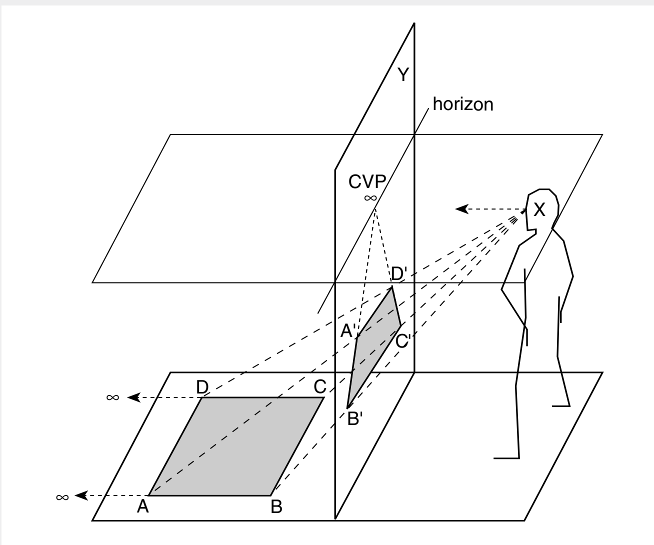


Alinari/Art Resource, NY

ed upon for his figures. A similar solution was adopted by Giotto in his "Confirmation of the Order of St. Francis," painted in 1325.

(Before proceeding further, the reader should consult Figure 13, for an introduction to the basic terminology of perspective drawing.)

FIGURE 13.



Above: The quadrilateral A′B′C′D′ is the cross section of the cone of vision; its base is ABCD, its apex X. When projected onto the Y plane, The ABCD square will become the A′B′C′D′ trapezoid. The central vanishing point is where the parallel lines AB and CD meet at a point in infinity, which will be one of the points on the horizon.

(a)

(b)

(c)

Left: Where one chooses to locate the horizon, depends upon where one has decided the viewer's standpoint shall be. Such subjectivity of standpoint thus has to do with how high, or how low the horizon line shall be.

If the horizon line is placed very low, as in (c), the sensation created is one of the landscape falling in upon us. In (a), we observe the landscape from above. Then, when figures are to be located in the landscape, their sightline must lie at the level of the chosen horizon if they are to appear to be in proportion to their surroundings.

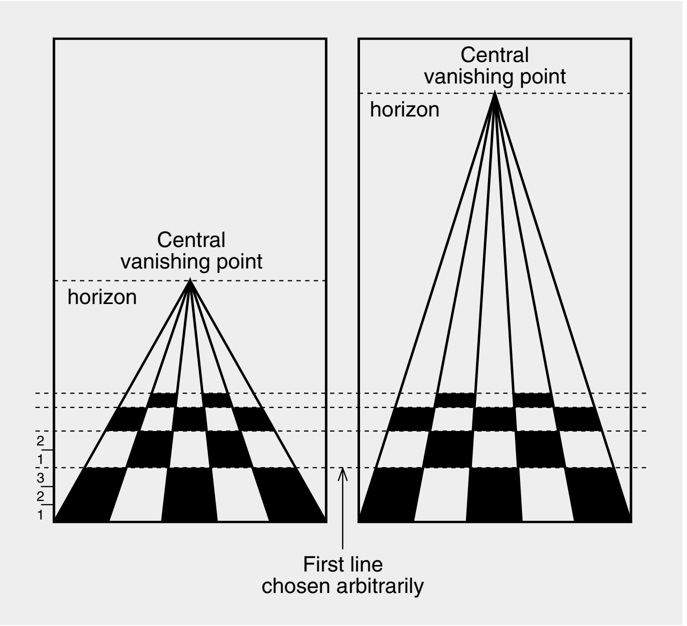


FIGURE 14. *Perspective according to the "musical system."*

Shortly thereafter, painters began to wonder as to how accurately defined receding distances might be pictured. Many simply ignored the problem, and continued to paint symbolic works. Others proposed, as a first approximation, the so-called musical system, according to which distances recede successively by thirds, two-thirds being the proportion proper to the musical interval of a fifth [SEE Figure 14]. Such a system, a mere arbitrary construct imposed upon reality, cannot possibly convey the notion of a harmonic whole. In order for something truly harmonious to be created, perspective must shift in accordance with the height of the horizon.

We owe the next step to the work of Donatello, Ghiberti, and Brunelleschi, sculptors and architects all. At the turn of the Fifteenth century, these three had been rivals in the great competition, by which Ghiberti was finally chosen to decorate the "Gates of Paradise" of the Florence Baptistery. It was they who first put to methodical use a second vanishing point, which they located not at the center, but at the side, of which system Ghiberti's *bas relief,* "The Story of Jacob and Esau," is a magnificent example [SEE Figure 15].

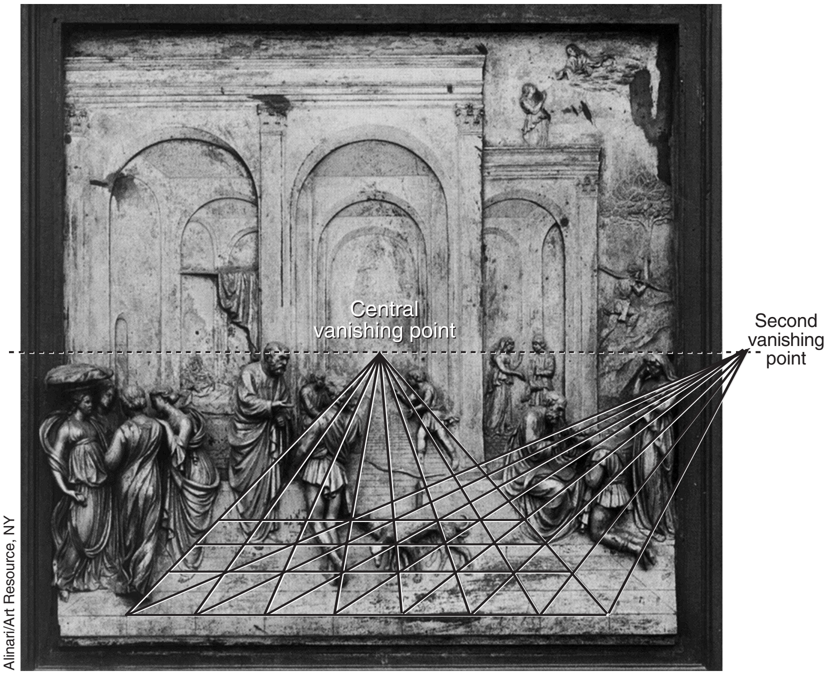
For the flagstones, Ghiberti chose a *braccio,* i.e., an arm's length, the convention of the time being that a man's height was generally three *braccia.* With the aid of these subtle reference points, Ghiberti drew a second figure in the background perfectly proportionate to that in the foreground.

It is greatly to be regretted that so few among the scientific treatises of that period, have come down to us; Paolo Toscanelli's *Della Prospettiva* (1420) is, to cite one notable example, lost. This mathematician and cartographer, friend to Cardinal Nicolaus of Cusa and to Brunelleschi, and mapmaker to Christopher Columbus, seems to have been a figure of the greatest importance to his age; had we his treatise still, we should doubtless have gained some considerable insight into the debate raging at the time over methods of perspective.

What *has* come down to us, is the well-known work of Leon Battista Alberti, *De Pictura* (1435); all the great breakthroughs in perspective, however, were made earlier, between 1401 and 1425, in which latter year Masaccio painted his fresco of "The Trinity." (Alberti came to Florence only in 1434, and could not have visited the city prior to 1428, when the ban exiling the Alberti family from Florence was lifted.)

As for Masaccio, it is believed that Brunelleschi himself helped to further his

FIGURE 15. *Ghiberti, "The Story of Jacob and Esau," Baptistery, Florence Cathedral, "Gates of Paradise" (1430-1437).*

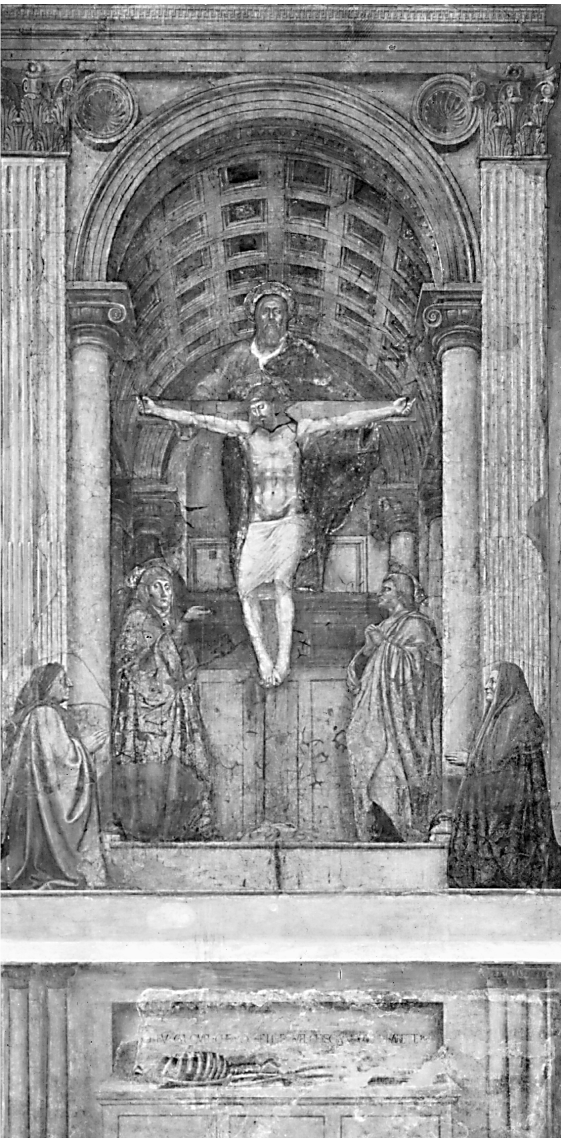


Alinari/Art Resource, NY

most extraordinary ability. It is often said, and with some reason, that "The Trinity" is the first true demonstration of perspective [SEE Figure 16]. The new science's great power is brought out by the "low angle" perspective: just below the foot of the cross lies the central vanishing point—there, exactly at eye-level, where the earthly and the heavenly worlds do separate.

Although dedicated to Brunelleschi, Alberti's *De Pictura* in fact defends Aristotle's doctrine of "*mimesis*": "Clearly, the painter has no concern for things that are not visible. And so, the painter is solely concerned to imitate the things which light shows us." Further on, Alberti quite adopts the axioms of Euclidean geometry, wherein points, lines, and surfaces are still, dead objects in a space made up of abstractions.

Neither in the Italian nor in the Latin text, does *De Pictura* delve at any depth into the fundamental issue of the horizon. In the final analysis, and notwithstanding the author's skill at weaving in the notion of a central vanishing point, nor the treatise's great importance in circulating this method beyond the guild workshop system, Alberti's work utterly contradicts the Renaissance principle, being an attempt to codify science in obedience to the standards of Aristotelean logic. To Alberti, the central vanishing point is a mere technical formula, not the principle of composition underlying a work of art. His method leads perforce to a *single* vanishing point, the lateral thereby becoming a mere aid to construction, which means that the painter has to keep within the framework of a symmetrical arrangement [SEE Figure 17]. (Early on, Alberti claims he will give mathematical proof that his system holds, but oddly enough, towards the end of Book II, we read:"It is my habit, when working with my closest collaborators, to adduce geometrical proofs in order to show in more perfect detail why these things are as they are, but I thought that such proofs might well be left out, so brief be my commentary here." Book II, 23.)



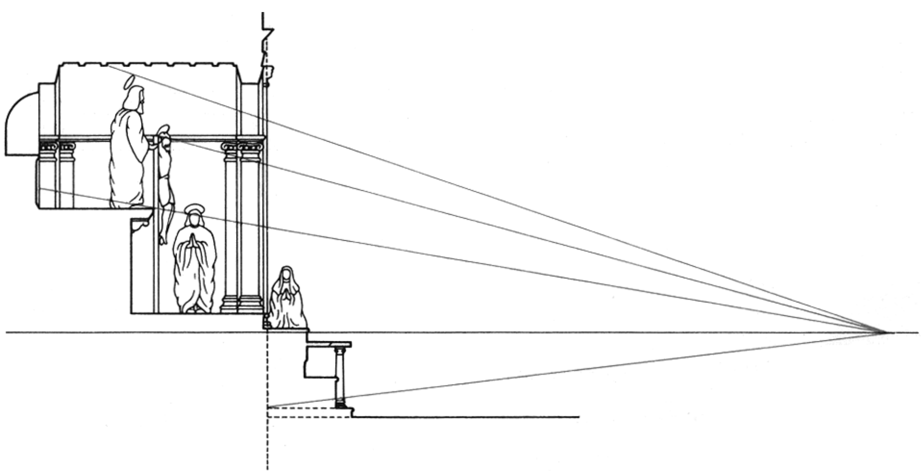


FIGURE 16. *Masaccio, "The Trinity" (1426).*

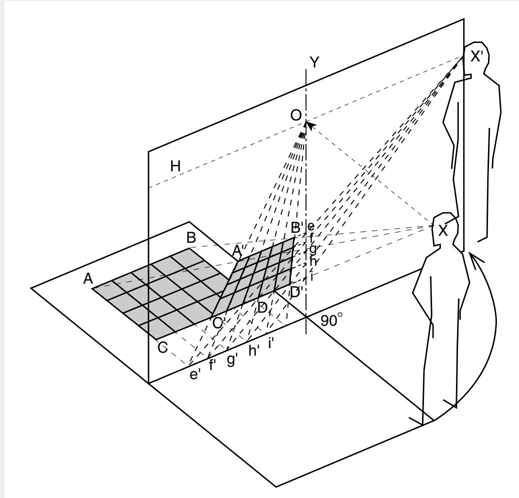


FIGURE 17. *Alberti's method. By making point X rotate 90° on the horizon (H), we obtain a second vanishing point (X′), one that is lateral, not central. By connecting the points e′f′g′h′i′ with O, we obtain the recession lines. If we connect them to X′, we get efghi on the intersection with the Y axis. By projecting efghi parallel to H, inside the triangle Oe′i′, we get the recession distances for the flagstones. If the drawing has been done accurately, the diagonals of the projected image will be straight lines.*

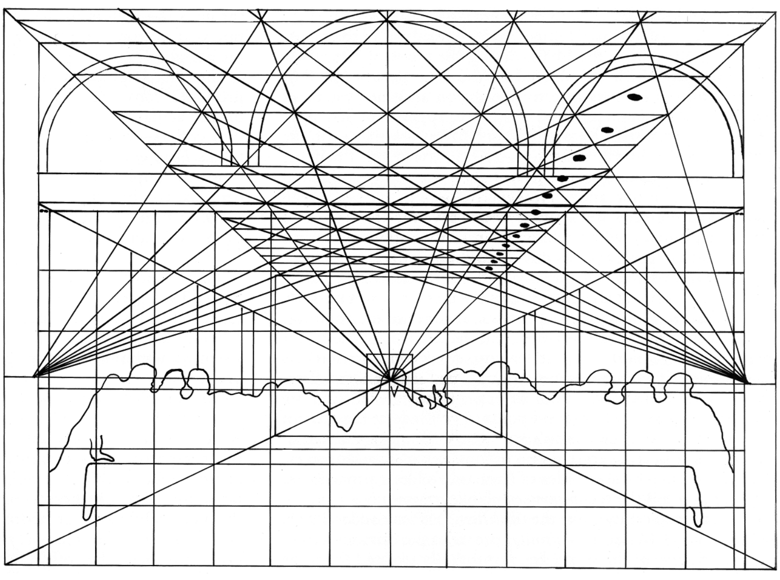
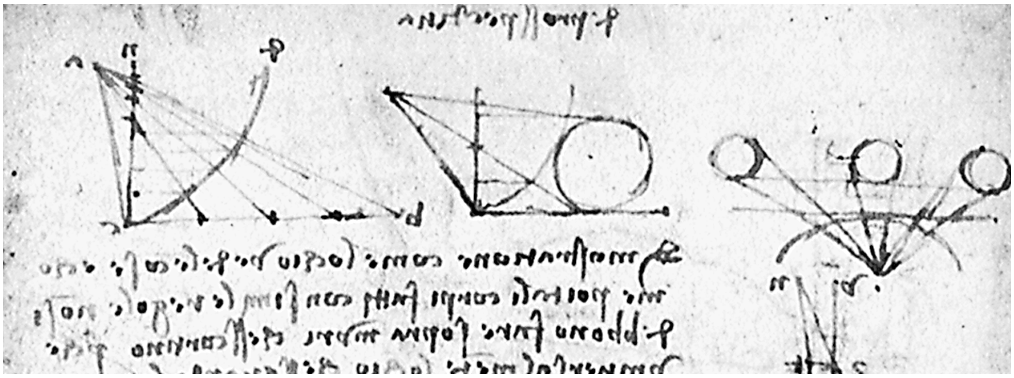


FIGURE 18. *Leonardo da Vinci, "The Last Supper" (1495-98).*

Diagram: The central vanishing point lies behind the head of Christ.

By 1450, Paolo Uccello, followed in this by Leonardo, had begun to explore a path which development of a second vanishing point had opened: by means of the second diagonal, one may arrive at a third vanishing point. In the Albertian system, the second diagonal was but a means to double-check the perspective; after 1450, it became the cornerstone of a new method, which did away with the complex projection heretofore used to plot distances as they recede on a surface. Alberti had placed these receding distances on the edge, or even, at times, far from the painting itself; thenceforth, all events fell within the field of vision.

How very great is the unifying potential of such a construction, is shown by Leonardo's celebrated "Last Supper": the central vanishing point lies behind the head of Christ, He, who has unified all Creation [SEE Figure 18]. It is at the the intersection of



Biblioteca Nacional, Madrid

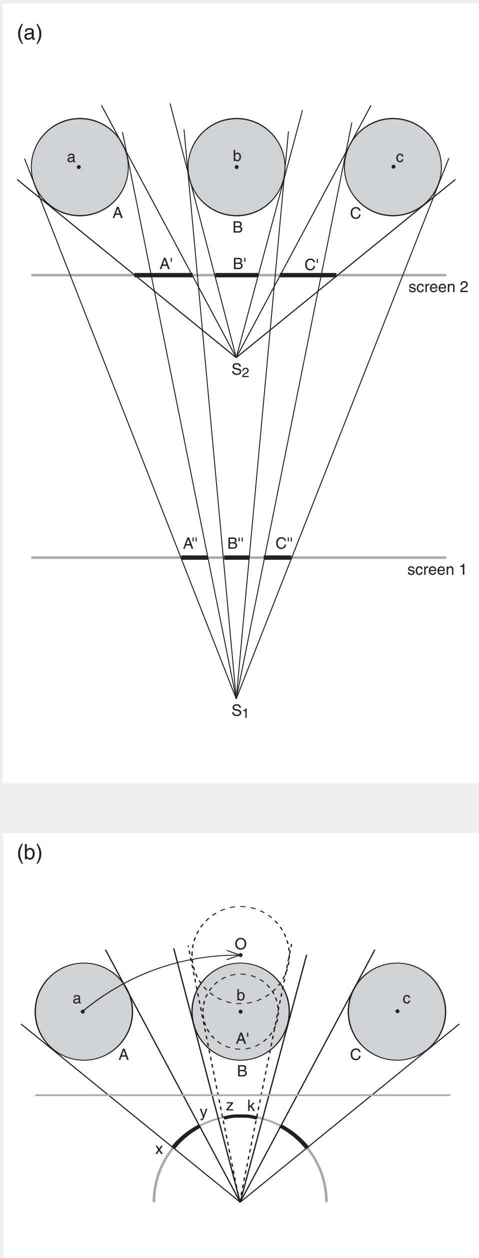
FIGURE 19. *Sketches by Leonardo. To the left can be seen one approach to curvilinear perspective, and to the right, Leonardo's famous paradox. (Madrid Codex II, folio 15v, detail)*

the three vanishing points that one finds the origin of the form of each singular element of the composition. Another advantage to this method, is that it opens the way towards asymmetrical compositions. Jean Pélerin Viator, once secretary to King Louis XI, was to put forward its merits in his *De Artificiali Perspectiva* (1505), printed at Toul in Eastern France; this was the first treatise on perspective ever printed in Europe.

Those who first defended this system were, not surprisingly, the first to find fault with it. According to some sources, Piero della Francesca points a finger in that direction in his *Di Prospectiva Pingendi* (1474). The other great trouble-maker was Leonardo himself, as we can see from the manuscript in the Madrid Codex, known as "The Paradox of Leonardo" [SEE Figures 19 and 20].

Linear perspective, as his Paradox shows, is but one of a number of possible cross sections of the visual cone. "Anamorphoses" is the name given to the representations of

FIGURE 20. *Leonardo's Paradox.*

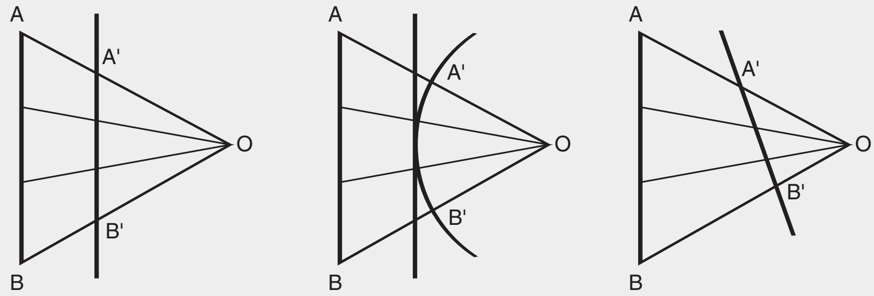


(a) If the spectator stands at S_1, and projects the image of three columns, A, B, and C onto screen 1, the projected image seems acceptable. If on the contrary, the spectator is at S_2 and projects the image onto screen 2, A′ will be bigger than B′, while A is further away from the spectator.

(b) Using the eighth theorem of Euclid's Optics, which postulates that the perception of distance is defined by the angle of vision, the columns' strict proportions can be restored by projecting their image onto a spherical surface. To verify this, we have rotated column A into the same angle of vision as B and have called it A′. Now, the projection of A′ onto the spherical screen is called zk and is equal to xy.

This paradox confirms Leonardo's insight into the limits of linear projection. The eye and its curved retina, as well as the rotation of the eyeballs, help man correct the distortions which otherwise increase, the nearer the eye is to the object.

Figure 21.
Cross sections of the visual cone.



other sections, whether elliptic, hyperbolic, or otherwise [SEE Figure 21]. These can be astonishing: in Holbein's painting "The Ambassadors," for example, the viewer must move, if he is to see the painting's "hidden" element, as the skull can only be seen when one stands at a tangent to the edge of the painting [SEE Figure 22]. The ambassadors stand before us, surrounded by all the attributes of the age's material wealth, its musical and its scientific instruments; yet "out of the corner of the eye" as it were, death steps in to disrupt the seeming quiet, recalling to our mind how ephemeral life is, and to what extent our senses trick us into forgetfulness. Once again, the artist has made us direct our gaze on a course which has to do with the composition's true, metaphorical meaning.

Anamorphoses thus bring out yet another shortcoming of linear perspective: there is but *one fixed point alone,* from which the viewer can really take in the painting.

Before turning to non-linear perspectives, let us examine one last example of linearity which is often mistakenly presented as an alternative to rectilinear perspective: curvilinear perspective. Striving to correct the tendency for space to be systematically deformed by linear constructions, the celebrated miniaturist Jean Fouquet, as well as a few of his contemporaries, worked out a curvilinear system. If one takes as a starting point,

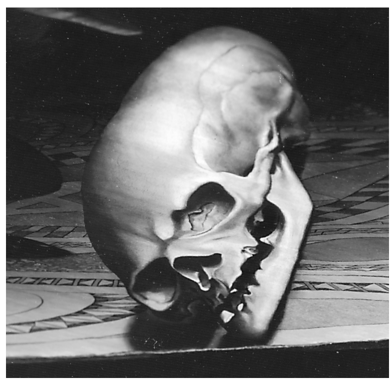
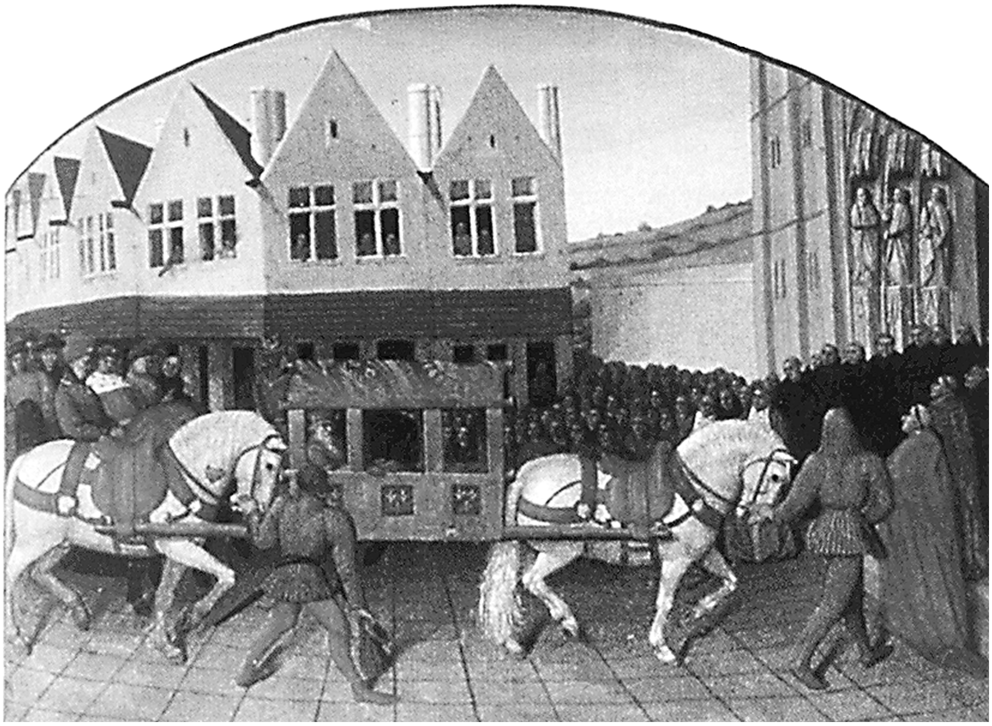


Figure 22. *Hans Holbein, "The Ambassadors" (1533). Detail, right: The skull seen from a tangent angle.*



Bibliothèque Nationale de France

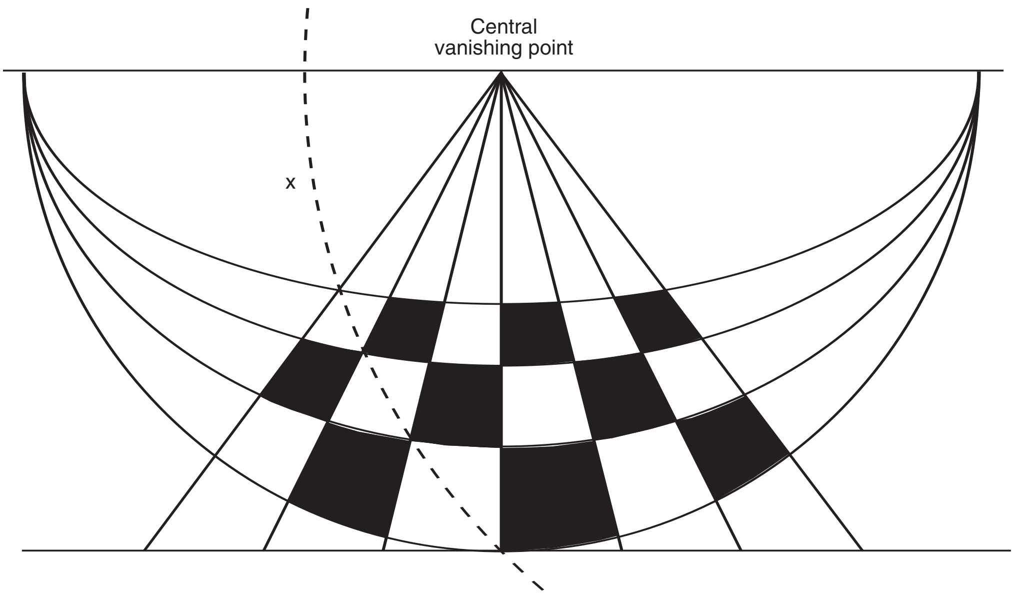


FIGURE 23. *Example of curvilinear perspective. Jean Fouquet, miniature of Charles IV arriving at the Cathedral of St. Denis (c.1458).*

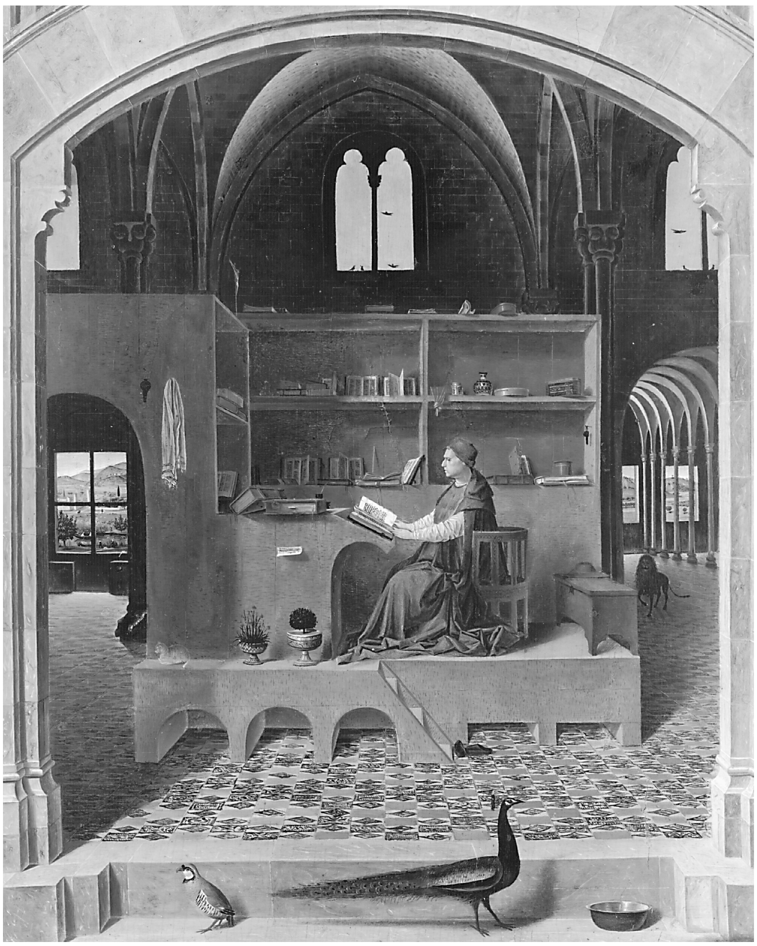
Diagram: The x-curve's crossing of the linear receding lines going from the baseline to the central vanishing point gives us the division of the distances.

the notion that distances should decrease to the viewer's left as well as to his right, the problem can, at least formally, be solved by tracing the arc of a circle [SEE Figure 23].

On the facsimile of the Madrid manuscript, Leonardo does adopt that method, although he seems to have been quite aware that at the end of the day, the problem is bound to pop up elsewhere: whether everything be made rectilinear, or again, curvilinear, one falls into the trap of one or the other structure which only a non-linear approach can pry open. Turner, very deliberately, and Van Gogh—(*cf.* the latter's "Bedroom in Arles")—more likely by intuition, began to explore the curvilinear path, which still holds out great promise.

Johannes Pötzsch

FIGURE 24. *Tilmann Riemenschneider, altarpiece, St.-Jacobskirche, Rothenburg (1500-1504) (detail of center panel).*



Reproduced by courtesy of the Trustees, The National Gallery, London

FIGURE 25. *Antonello de Messina, "St. Jerome in His Study" (c.1474) (detail).*

Non-Linear Perspective

The non-linear approach, far from being a thing of recent invention, has always co-existed with the linear, the two having developed in symbiosis, and in complementary opposition. Where linearity seeks to *unify,* albeit at the *expense* of the manifold, non-linearity rests upon the persuasion, that it is only through the greatest possible unfolding of the Many, that there shall be attained a Unity, greater even than that which may be depicted. It being the case, that the mind will tend to confuse unity, with *uniformity.*

I may choose, in order to further the unity of a composition, to ignore a detail, or allow it to fade away. Or, I may choose to bring out the beauties and the profuseness of the Many, by showing the degree to which they partake in Unity. In music, several chords may be made to vibrate at once: musical unity is not at all the same as unison, but has to do with harmonic composition, to which dissonant elements also belong. So it is with space, which must be made to live, and from which all that gives off a sense of cold and void should be expelled. A detail, seemingly minute, a window, may let the spirit escape into the infinite.

One *tour de force* of this kind, might be named the *perspective of suggested space.* When depicting, let us say, a loggia, one may, by letting in windows or adjacent hallways, suggest other spaces without ever drawing them in. When, for his altarpiece at the Church of St. Jacob in Rothenburg, Tilman Riemenschneider sets tiny bright panes of glass into his sculpture, it is all the more remarkable for the fact that one has gone from the texture of wood to that of glass [SEE Figure 24].

As regards this principle in painting, let us look closely at Antonello de Messina's "St. Jerome in his Study" [SEE Figure 25]. The viewer finds himself standing before a house, into the rooms of which he may gaze, while, through its windows, he further perceives a far-off landscape. Thus, while St. Jerome is shown in the privacy of his study, yet we see him as if in an open space. By allowing our gaze to light upon a succession of spaces, each unlike the next, Antonello introduces a sense of greater freedom.

What might be called *narrative perspective,* pertains to the same school of thought. Space is built up by a succession of all manner of elements, the proportions of which can only with difficulty be appreciated. In Jan van Eyck's "Virgin of Autun" ("The Virgin with Chancellor Rolin"), there is a loggia in linear perspective, beyond which and a little below it, a garden is to be seen; at the garden's edge stand ramparts, from which two men look down upon a river meandering towards a bridge; and over the bridge, wind a great many tiny figures and seven, or perhaps more, horses; further still in the distance, in a bend of the river, rises a castle; and behind it, snow-capped mountains; and so it goes . . . . [SEE Figure 26] Although this construction is not a mathematical one, we are yet led, by the manner in which each succeeding plane somehow telescopes into the next, to experience space as a *discontinuous whole.* At the end of the day, the loggia may well be found to lie at a celestial height, which effect the painter appears precisely to have sought, for it is in the meeting between the mortal and the divine that the scene's true meaning lies.

Yet another sort of non-linear perspective is that known as the *dancing horizon.* Rather than a single horizon, why should there not be several?

It is, after all, the mind which "builds" a perspective, wherever we choose to cast our eye. The most brilliantly successful, and least understood, example of this is the "Mona Lisa" [SEE Figure 27]. Own up!

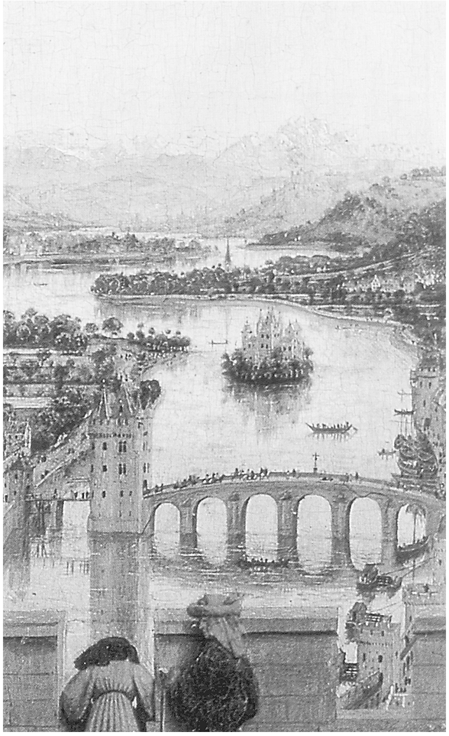


FIGURE 26. *Jan van Eyck, "The Virgin of Autun" ("The Virgin with Chancellor Rolin") (1436). Left: Detail showing "narrative perspective."*

blockade sealing off the city.

Light—above all, in the case of Rembrandt—was to become an extraordinarily powerful means to suggest the existence of spaces not explicitly shown. In Rembrandt's work, there is dialogue between the light within, and light from without; what cannot be pinpointed, is the source of such light. There is thereby conveyed a most powerful impression of how the presence of an individual being, effects the transformation of light [SEE Figure 29]. It is Light itself, therefore, which has become the new Transfinite, and Rembrandt, in this particular respect, shows himself to have been a true disciple of Leonardo.

To Leonardo, a limit is defined, not by a line as such, but as a change in the geometry or sense of orientation. *Sfumato,* a technique through which one consciously blurs or softens a figure's outline, is a first step toward defining the material world in terms of a higher reality, Light. Is it not through light, and light alone, that we see? And is it not the play of light and shadow, which shapes what we see?

Color, and how color evolves through space, is to Leonardo yet another means to free the composition from linear con-

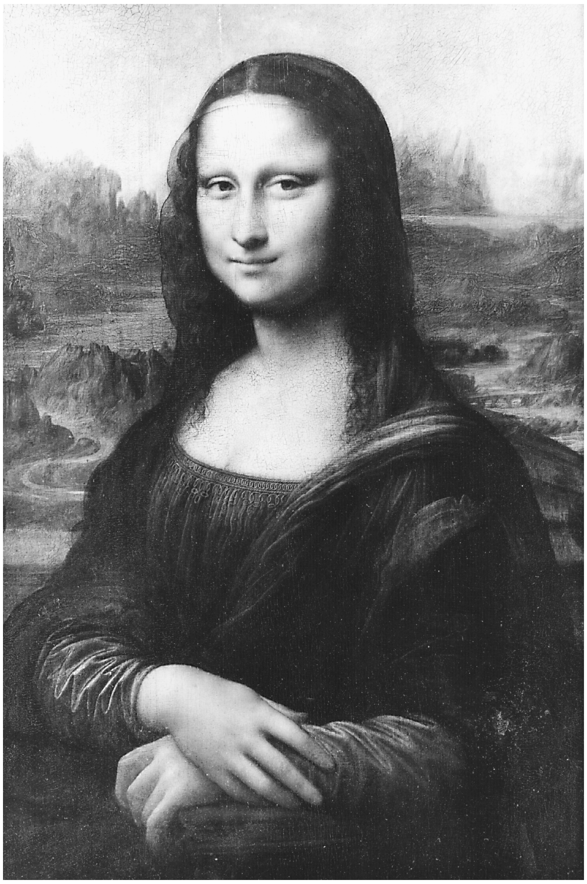


FIGURE 27. *Leonardo da Vinci, "Mona Lisa" (1503).*

Had you really swallowed Freud's fraudulent tall tale about Leonardo, the transvestite, disguising your mother-in-law's nasty smirk behind the lovely lady's smile? The fact is, that what unsettles us is not the lady herself, but the landscape beyond. To the left of her face, the horizon lies more or less at the level of the nose, while to the right, a horizon appears to float somewhere about the level of the eyes. As one goes on studying the painting, other horizons swim into view.

We find a similar procedure in "The Siege of La Rochelle," by Jacques Callot, the noted engraver from Nancy [SEE Figure 28]. Whether frontal or from above, the views are integrated into the self-same plane. Here again, one may imagine a series of horizons, ranging from those which, as we examine the foreground, lie rather low, to those fading off into the far distance as we study the naval

FIGURE 28. *Jacques Callot, "The Siege of La Rochelle, 1627" (detail) (1631).*

straints. In his own words: "In nature, the perspective of color obeys her laws always, whereas, that of magnitude is arbitrary: next to the eye, there may lie a little hill, and far off in the distance, a great mountain . . . ." (Manuscript A, Institute de France, folio 105v) To the extent that we cling to the domain where forms be represented as such, we may be deceived by what we think we see; on paper, a tiny but proximate object looks as large as a great one that lies very distant from us. Hence, Leonardo's work on *aerial (atmospheric)* or *color perspective,* which he describes thus:

> There is another kind of perspective which I call Aerial Perspective, because by the atmosphere we are able to distinguish the variations in distance of different buildings, which appear placed on a single line; as, for instance, when we see several buildings beyond a wall, all of which, as they appear above the top of the wall, look of the same size, while you wish to represent them in a picture as more remote one than another and to give the effect of a somewhat dense atmosphere. You know that in an atmosphere of equal density the remotest objects seen through it, as mountains, in consequence of the great quantity of atmosphere between your eye and them—appear blue and almost of the same hue as the atmosphere itself when the sun is in the East. Hence you must make the nearest building above the wall of its real color, but the more distant ones make less defined and bluer. Those you wish should look farthest away you must make proportionately bluer . . . . (Ashburnham I, folio 10a)*

(The reader should note, that the word "aerial" here has its original meaning of "airy" or "pertaining to air" and its gradations (i.e., "atmospheric"); it does not mean, as it would in contemporary acceptance, "seen from above.")

In this manner, we begin to leave behind formal perspective, wherein objects have characteristics, such as magnitude or color, which are fixed, and move rather towards a physical perspective, where the changes Leonardo speaks of are taken into account, according to the subjective conditions of where the object is to be located. In other words, objects, or the elements of a landscape, are painted taking into account their physical interactions, which interactions had lain almost entirely outside the field of linear perspective. Space has ceased to be an empty place, to become a field of interaction. What here transcends the subjective aspect, are the actual physical principles at work.

FIGURE 29. *Workshop of Rembrandt van Rijn, "Portrait of Rembrandt" (1650).*

It is from our awareness of those principles that there springs a sense of having seen, not reality as such, but rather Truth, in Leonardo's painting; for, these principles, which we recognize as underpinning the universe, pertain more to *truth* than to *reality.* In this manner, Leonardo proves that artistic beauty and scientific knowledge are truly one.

It is also Leonardo who introduced the notion of *fading perspective,* or *perspective of disappearance*:

> Every object as it becomes more remote loses first those parts which are smallest. Thus of a horse, we should lose the legs before the

---

* *The Notebooks of Leonardo da Vinci, compiled and edited from the original manuscripts by Jean Paul Richter* (New York: Dover Publications, 1970), Vol. I, p. 159, No. 295.

Important Dates in the Invention of Perspective

The Greek Classical Age

Fifth-century B.C.

- **Agartharcus, Anaxagoras, and Democritus**. In his *De Architectura* (Vol. III, Bk. 7), the Roman architect Vitruvius writes ". . . Agarthacus, in Athens, when Aeschylus was bringing out a tragedy, painted a scene, and left a commentary about it. This led Democritus and Anaxagoras to write on the same subject, showing how, given a center in a definite place, the lines should naturally correspond with due regard to the point of sight and the divergence of the visual rays, so that by this deception a faithful representation of the appearance of buildings might be given in painted scenery, and so that, though all is drawn in a vertical flat façade, some parts may seem to be withdrawing into the background, and others to be standing out in front." (Para. 11)
- **Plato**, in the *Sophist,* condemns the sculptors' fascination with illusion.
- Plato's *Timaeus* dialogue deals with the problem of what appears to be an opposition between emission and reception of visual "radiation": "When, therefore, the daytime light surrounds this stream of vision, then like meets like, both fusing together, and one homogeneous body is formed along the line of vision wherever the light from inside the eyes encounters some external object. And so the whole stream of vision, because of its similarity, is similarly affected, so that if it ever touches some objects or is touched by them, it passes on the movements from these throughout the whole body right into the soul, and causes the sensation we call seeing." (Steph. 45c)
- Pliny the Elder praises the illusions of space painted by **Zeuxis, Parrhasius, and Apollodorus**.

Third-century B.C.

- **Archimedes** writes that "the eyes do not see from a single point, but from a certain magnitude," thus anticipating a solution to "Leonardo's Paradox."
- **Euclid**, *Optics and Catoptrics*.

The Modern Age and Golden Renaissance

Eleventh-century A.D.

- **Al-Hazen** writes *Optics* and *On geometrical curvature* (treatises).

Thirteenth-century A.D.

- **1265: Nicola Pisano**, sculptor, active at Pisa and Siena.
- **1267:** Franciscan monk **Roger Bacon** writes his *Opus Majus*.

Fourteenth-century A.D.

- **1325: Giotto** paints "The Confirmation of Saint Francis" in Florence.
- **1333: Simone de Martini** paints "The Annunciation."
- **1342: Pietro Lorenzetti** paints "The Birth of the Virgin" in Siena.
- **1375:** Birth of **Robert Campin**, the Master of Flémalle. He was to work for the Carthusian monastery at Champmol, near Dijon, the capital of Burgundy. He taught Rogier van der Weyden, and greatly influenced Jan van Eyck.
- **1376:** Founding of the teaching order of the **Brothers of the Common Life** in Deventer (The Netherlands).
- **1385:** Dutch sculptor **Claus Sluter** completes the fountain, now known as the "Moses-well," at the Champmol monastery.

Fifteenth-century A.D.

- **1401:** Competition at Florence to decide who shall execute the *bas reliefs* for the Baptistery's second Gate.
- **1410-24: Brunelleschi**, as *per* notes written by Antonio Manetti around 1475, tests his perspective constructions against reality, by looking through a small hole in a painting, towards the image of the Baptistery reflected onto a looking glass. Manetti does not however say how the perspective drawing should be carried out.
- **1420: Paolo Toscanelli** writes *Della Prospettiva* (treatise). Works with Brunelleschi; the latter takes up the challenge to complete the Cathedral's cupola, a thing believed to be impossible at the time.
- **1423: Nicolaus of Cusa** stays in Padua, where he probably meets his friend Toscanelli.
- **1423: Donatello** sculpts "Herod's Feast" for the Baptistery door at Siena, with a vanishing point perspective.
- **1426: Masaccio** paints "The Trinity," at Santa Maria Novella in Florence.
- **1432: Jan van Eyck** paints "The Mystic Lamb," altarpiece for the Cathedral at Ghent (modern Belgium).
- **1435: Ghiberti** completes the Gates of the Florence Baptistery, after thirty-four years of work.
- **1435: Leon Battista Alberti** writes *De Pictura,* his treatise dedicated to Brunelleschi.
- **1436: Van Eyck** paints "The Virgin of Autun" ("The Virgin with Chancellor Rolin"). That same year, he paints the French composer Guillaume Dufay in "Timotéos." Dufay composed a four-voice motet, sung in the Cathedral of Florence during the Council.
- **1437-39: The Council of Ferrara**, later removed to Florence to flee the plague, adopts the "*Filioque*."
- **1445: Ghiberti** writes his *Commentaries*.
- **1460: Jean Fouquet** paints miniatures in *The Book of Hours of Etienne Chevalier*.
- **1474: Piero della Francesca** writes *De Prospectiva Pingendi* (treatise).
- **1492: Christopher Columbus** reaches the New World, guided by a map drawn by Toscanelli, which suggested that a path to the Indies lay to the west.

Sixteenth-century A.D.

- **1503: Leonardo da Vinci** paints the "Mona Lisa."
- **1505: Jean Pélerin Viator**, once a secretary of France's Louis XI, writes *De Artificiali Perspectiva,* the first printed treatise on perspective in Europe.
- **1509-1511: Raphael** paints "The School of Athens" in the Vatican.
- **1518: Raphael** paints "The Transfiguration."
- **1525: Albrecht Dürer** writes his *Underweysung der Messung* (*Instruction on Measurement*).
- **1533: Hans Holbein** paints "The Ambassadors."

> head, because the legs are thinner than the head; and the neck before the body for the same reason. Hence it follows that the last part of the horse which would be discernable by the eye would be the mass of the body in an oval form, or rather in a cylindrical form and this would lose its apparent thickness before its length . . . . (Manuscript E, Institute de France, folio 80b)†

This means, that the greater the distance between the eye and the object it observes, the more do the outlines of that object fade. And of an object overly close, the same may be said:

> When an object opposite the eye is brought too close to it, its edges must become too confused to be distinguished; as it happens with objects close to a light, which cast a large and indistinct shadow, so is it with an eye which estimates objects opposite to it; in all cases of linear perspective, the eye acts in the same way as the light. (Manuscript A, Institute de France, folio 103v)‡

A fine illustration of the above, is a painting attributed to Rembrandt, "The Philosopher" [SEE inside front cover, this issue]. In the foreground, we discover objects the outlines of which are blurred. Note how this technique accelerates the impression of depth and light. Clearly, Leonardo had struck gold: the way we perceive space is, indeed, defined by light alone, and by the manner in which light leads us to confront the universe.

As we come to the end of this study, let us linger a moment on Pieter Bruegel the Elder's painting, "The Magpie and the Gallows" [SEE inside back cover, this issue]. A marvellous landscape stretches before us, painted in accordance with Leonardo's rules for aerial perspective. Oblivious to that vaster plane, their sight hindered by trees and thick hedgerows, rural bumpkins dance about, only to perish somewhere between the cross and the gibbet. —May this not tell us something of the purpose of sight in our own lives?

Translated from the French
by Katherine Kanter

---

† *Notebooks,* Vol. I, p. 127, No. 223.
‡ *Notebooks,* Vol. I, p. 57, No. 92.

BIBLIOGRAPHY

(English-language editions of works originally cited in French have been supplied wherever possible.)

Leon Battista Alberti, *On Painting,* trans. by John R. Spencer (New Haven: Yale University Press, 1966).
Aristotle, *Poetics* (New York: Dover, 1951).
Jurgus Baltrusaitis, *Anamorphoses,* trans. by W.J. Strachan (New York: Harry N. Abrams, 1977).
Roland Bechmann, *Villard de Honnecourt* (Paris: Picard, 1993).
L. Brion-Guerry, *Jean Pélerin Viator et sa place dans l'histoire de la perspective* (Paris: Les Belles Lettres, 1962).
Jacques Cagliardi, *La conquête de la peinture* (Paris: Flammarion, 1993).
Jacques Cheminade, "Lazare Carnot, organisateur de la Nation," Speech at Beaune, France, November 1989; published in: *Regard sur la France républicaine* (Paris: Editions Alcuin, 1991).
Hubert Damisch, *The Origin of Perspective,* trans. by John Goodman (Cambridge, Mass.: MIT Press, 1994).
Leonardo da Vinci, *The Notebooks of Leonardo da Vinci* (New York: Dover, 1970).
______*Treatise on Painting,* trans. by A. Philip McMahon (Princeton, N.J.: Princeton University Press, 1956).
Albrecht Dürer, *The Painter's Manual,* trans. by Walter L. Strauss of *Underweysung der Messung* (New York: Abaris Books, 1977).
Euclid, *L'Optique et la catoptrique* (Paris: Blanchard, 1959).
Alfred Fäustle *Perspektief en Projektie* (de Bilt: Cantecleer, 1971).
John Frisby, *Seeing: Illusion, Brain, and Mind* (New York: Oxford University Press, 1979).
Robert Gallagher, "Leonardo da Vinci's Science of 'prospettiva,'" *21st Century Science & Technology,* Vol. 2, No. 6, Nov.-Dec. 1989.
Histoire de l'Art: 1000-2000 (Paris: Hazan, 1995).
Martin Kemp, *The Science of Art* (New Haven: Yale University Press, 1990).
L'Italie de la Renaissance, un monde en mutation (Paris: Fayard, 1990).
Lyndon H. LaRouche, Jr., "On the Subject of Metaphor," *Fidelio,* Vol. I, No. 3, Fall 1992.
Roy McMullen, *Mona Lisa: The Picture and the Myth* (Boston: Houghton Mifflin, 1975).
Peter and Linda Murray, *The Art of the Renaissance* (New York: Oxford University Press, 1963).
Erwin Panofsky, *Early Netherlandish Painting: Its Origin and Character* (Cambridge, Mass.: Harvard University Press, 1953).
Paul Philippot, *La peinture dans les anciens Pays-Bas* (Paris: Flammarion, 1994).
Les Primitifs flamands et leur temps (Louvain: La Renaissance du Livre, 1994).
"Plato's Timaeus; the only authentic English translation," The Campaigner, Vol. 13, No. 1, Feb. 1979.
Oliver Sacks, *The Man Who Mistook His Wife for a Hat, and Other Clinical Tales* (New York: Perennial Library, 1987).
Pierre Thuillier, "La naissance de la perspective," *La Recherche (Paris),* No. 160, November 1984.
Vitruvius, *The Ten Books of Architecture* (New York: Dover, 1960).
Tjomme de Vries, *Het tekenen van de ruimte, toepassingen van de perspectief* (Utrecht/Antwerp: Het Spectrum, 1966).