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Mozart's 1782-1786 Revolution

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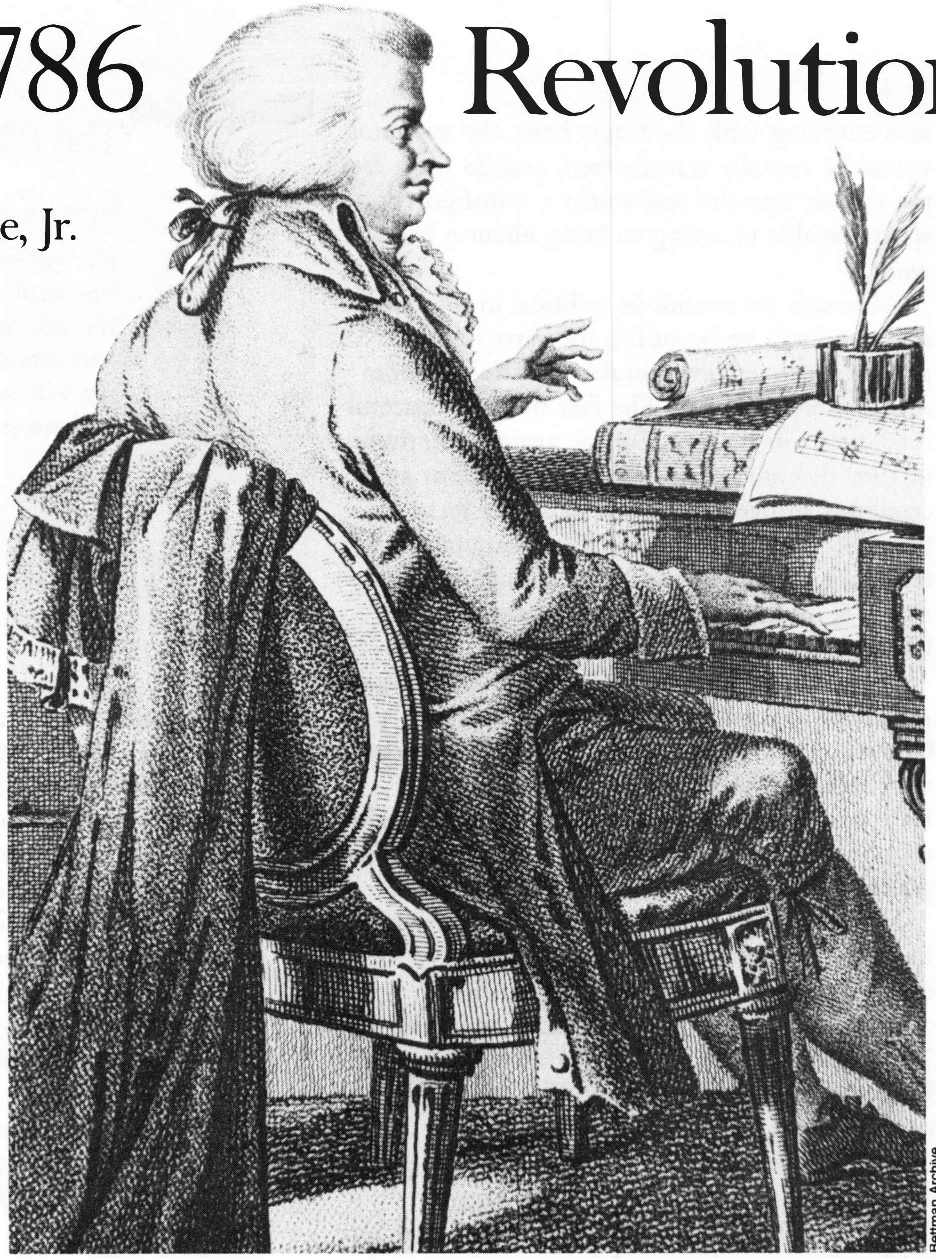


I.

Contrary to widespread, illiterate custom, the word "Classical," when employed in its strictest, epistemological sense, signifies any species of fine-arts composition which coheres with Plato's principles for aesthetics.[1] More recently, all of the development of modern Classical polyphony, from Florence, Italy of the early fifteenth century, through the 1896 Johannes Brahms' composing his "Four Serious Songs," defines—as we have noted elsewhere—a corresponding phase of musical progress to be of a specific *Cantor Type*.[2] In this following review of a forthcoming musical textbook,[3] we shall focus upon a still narrower interval of time, the crowning accomplishment in all musical development to date, that century-odd development of Classical polyphony which began with Joseph Haydn's revolutionary six "Russian" string quartets, Opus 33, of 1781. We concentrate here upon a crucial facet of that three-fold, Haydn-Mozart musical revolution of 1781-1786, which began the ensuing hundred-odd years of progress.

This revolution of 1781-1786 combines three distinct revolutions into one. Each of these three is defined as a "revolution" in its own right, in the same sense we attribute that quality to a valid discovery of principle in physical science.[4] Taken in order of their impact upon Wolfgang Amadeus Mozart, these three revolutions are as follows. The first in this sequence, is Haydn's discovery of his *Motivführung* principle of composition, as this is represented by his 1781, Opus 33 string quartets.[5] The second, is Johann Sebastian Bach's 1747 *Musical Offering*.[6] The third, is Mozart's insight into the integration of these two preceding discoveries by Haydn and Bach. Mozart's discovery is represented immediately by a series of his compositions from the 1782-1786 interval. Among the most notable of these latter, are his six "Haydn" string quartets (K. 387, 421, 428, 458, 464, 465), his C-

in Music

minor Mass (K. 427), his keyboard fantasy-sonata K. 475/457, and his celebrated keyboard concerti in D-minor (K. 466) and C-minor (K. 491).

The characteristic feature of this 1781-1782 Haydn-Mozart revolution, is the successful development of a principled new conceptual approach to Classical composition, an approach by means of which a complete work—such as a theme with variations and fugue, or a sonata, or a symphony, or a concerto, or a string quartet—might achieve that singular perfection of unity of effect which is the subject of Plato's *Parmenides* dialogue, the dialogue on the matter of "the One and the Many."[7] The subject of this following review is a crucial aspect of that three-fold revolution of 1781-1786, the relation of those discoveries to the principle of "Platonic ideas." That aspect is identified by the term "musical thought-object."

That Haydn-Bach-Mozart revolution is the underlying, unifying theme of the forthcoming, second volume of a two-volume musical textbook, *A Manual on the Rudiments of Tuning and Registration.*[8] Volume I, a Fall 1992 release, covers, principally, tuning and the registration of the *bel canto*-trained species of polyphonic singing voices. The second volume, for 1993 release, treats the circa 1815-1849 perfection of the Classical chest of orchestral and keyboard instruments,[9] from the standpoint of *bel canto* vocal polyphony. This second volume uses Beethoven's integration of soloist, chorus, and orchestra in his *Missa Solemnis* and *Ninth Symphony* as benchmarks for portraying the overall development of the Classical performing medium during the period from Handel and Bach through Brahms' work.

Once the 1781-1786 *Motivführung* revolution had been established, by Haydn, Mozart, and then Beethoven, the polyphonic medium of performance must be brought into conformity, in form and application, with the requirements of that new principle of composition. The pivotal instrumental feature of the required congruence, is the evolved string quartet of Haydn, Mozart, Beethoven, *et al.*: two violins, viola, and 'cello. This combination

Left: Wolfgang Amadeus Mozart.

is a musical medium in its own right, but also the kernel of the Classical chest of orchestral instruments.

To make this connection clearer to the non-professional: each species of *bel canto* singing voice (soprano, mezzo-soprano, contralto, tenor, baritone, bass) is distinguished from the others by its own, unique, spectroscopic set of register-passing frequency-bands (SEE Figure 1). Each string of each species of string instrument is an available surrogate for some individual register of a species of singing voice (SEE Figure 2). Thus, if a composer assigns the part of a soprano voice to the first violin, a mezzo-soprano to the second violin, a tenor to the viola, and a bass to the 'cello, the performer need but pass to a different register (string) on the appropriate choice of register-passing tone (SEE Table I).

However—to continue to the next step of this illustration—by changing the register-passages of an instrument in the relevant fashion, the performer can imitate the registral spectroscopy of any species of singing voice—although, often, in a vocal range displaced from that of the singer (SEE Figure 3). In contrast to this facility of the strings, wind instruments (SEE Figure 4) have essentially fixed registral characteristics, each corresponding to a specific choice of singing-voice species. Thus, the use of the polyphonic principle perfected by Handel, Bach, Haydn, Mozart, *et al.,* implicitly requires greater emphasis upon the highly developed form of string ensemble, centered upon the string quartet, as the keystone of the Classical orchestra. The Haydn *Motivführung* principle, as apprehended by Mozart, takes us to the heart of this challenge for development of the appropriate approach to composition for the orchestra.

Consider an illustration of this point from Mozart's 1782 C-minor Mass, K. 427 (SEE Box, p. 10). The violin here is imitating the soprano singing voice, but at a displaced range. The point is illustrated in another respect, by studying cases of Mozart's and Beethoven's transcriptions for strings of some of their own earlier compositions for wind instruments.[10] The string quartet, augmented by the double bass, generates an orchestral chest of stringed instruments which maps super-densely the entire vocal polyphonic range, and freely extends it for every species of actual or imaginable spectroscopic species of singing voice. The relationship between these stringed choruses and the soloist-like wind instruments, is the key to the evolution of the orchestra, especially from 1781-1782 onward, an orchestra suited to the implied requirements and potentialities of the *Motivführung* revolution.

The Root of the *Motivführung*

In a general way, any person steeped in the Classical polyphonic repertoire should recognize, as if by reflex, many among the leading musical points considered in this review. Even if such a person did not know the crucial circumstances of Haydn's revolutionary Opus 33, certain relevant points are abundantly clear to the same effect from simple observation. The person should be aware of a certain kind of superiority of coherence appearing more and more in the later string quartets, sonatas, symphonies, and concerti of Joseph Haydn, and those of Mozart, both in comparison with the relevant work of the Scarlattis, Handel, Bach, and Bach's famous sons. There is visible to that same effect, a striking, revolutionary change toward much greater coherence, in Haydn's composition, beginning his Opus 33. A study of Haydn's own work of the 1763-1782 interval, and also a comparative study of Mozart's work over the 1773-1786 interval, brings the point into clearer focus.

One of the contributing scholars for Volume II of *A Manual on the Rudiments of Tuning and Registration* suggested the following special studies be included. In addition to the obvious comparison of Haydn's Opus 33 with his 1771 Opus 20, "Sun" quartets, compare his 1771/73 Symphony No. 52 with the 1782, more "Bachian" Symphony No. 78. Look back to the Fourth Movement of his 1765 Symphony No. 13; compare this not only with his Symphonies No. 52 and 78, but with the Finale of Mozart's 1787 ("Jupiter") Symphony No. 41.

Such comparisons show a persisting, developing effort, in the pre-1781 compositions, to master a stubborn paradox. Suddenly, with the Opus 33, the discovery, the solution bursts into view, as is the case for a valid major discovery in physical science. This Haydn discovery leads Mozart to recognize the special import of an earlier discovery, the *Musical Offering,* by Bach, with the resulting general consequence identified. This process has an eerie resemblance to the most crucial discovery of the Golden Renaissance's founding of modern physical science: Nicolaus of Cusa's discovery of his own "isoperimetric" solution[11] for Archimedes' profoundly paradoxical efforts to define a square whose area is equal to that of a given circle.[12] There is a connection between Cusa's

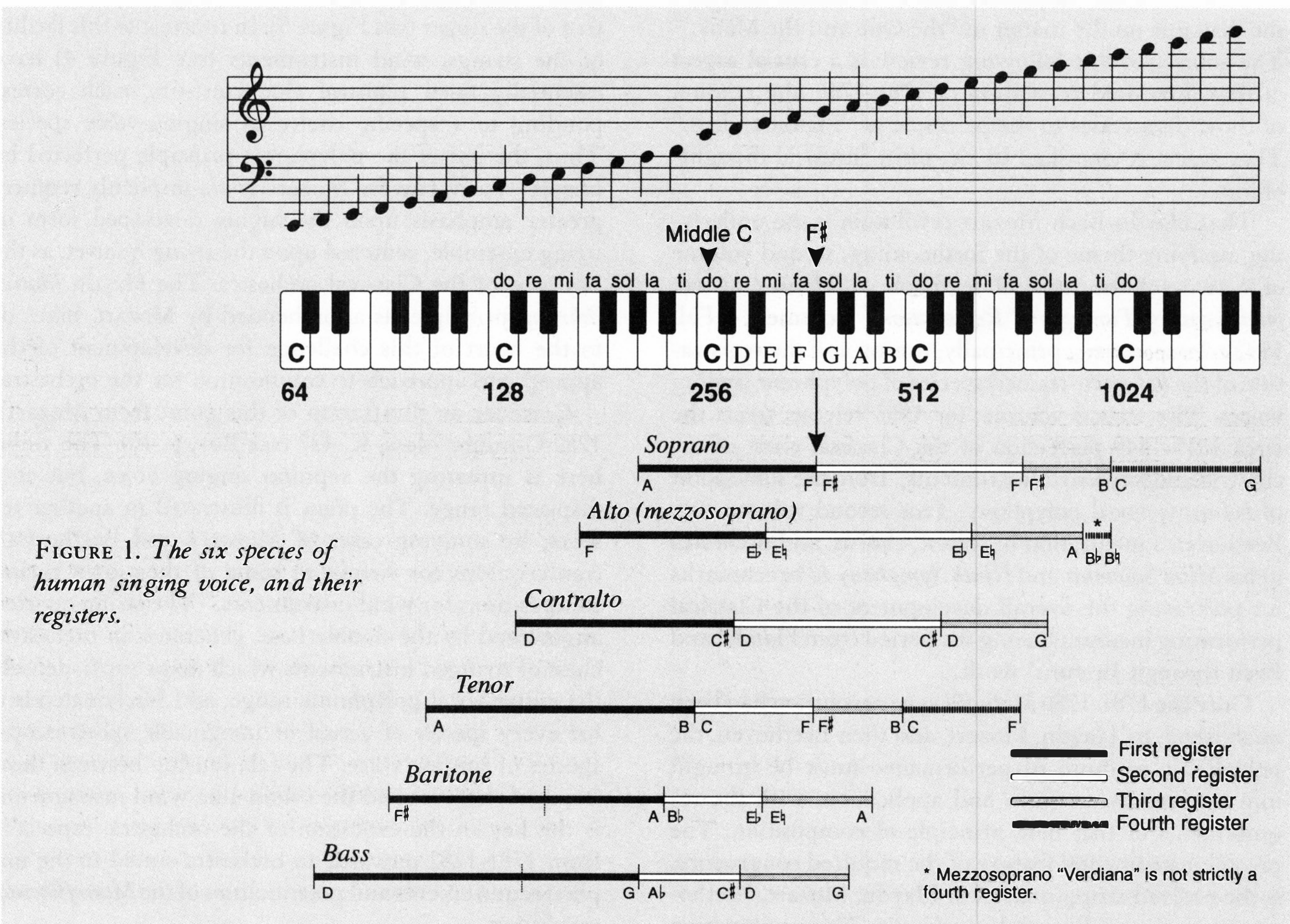


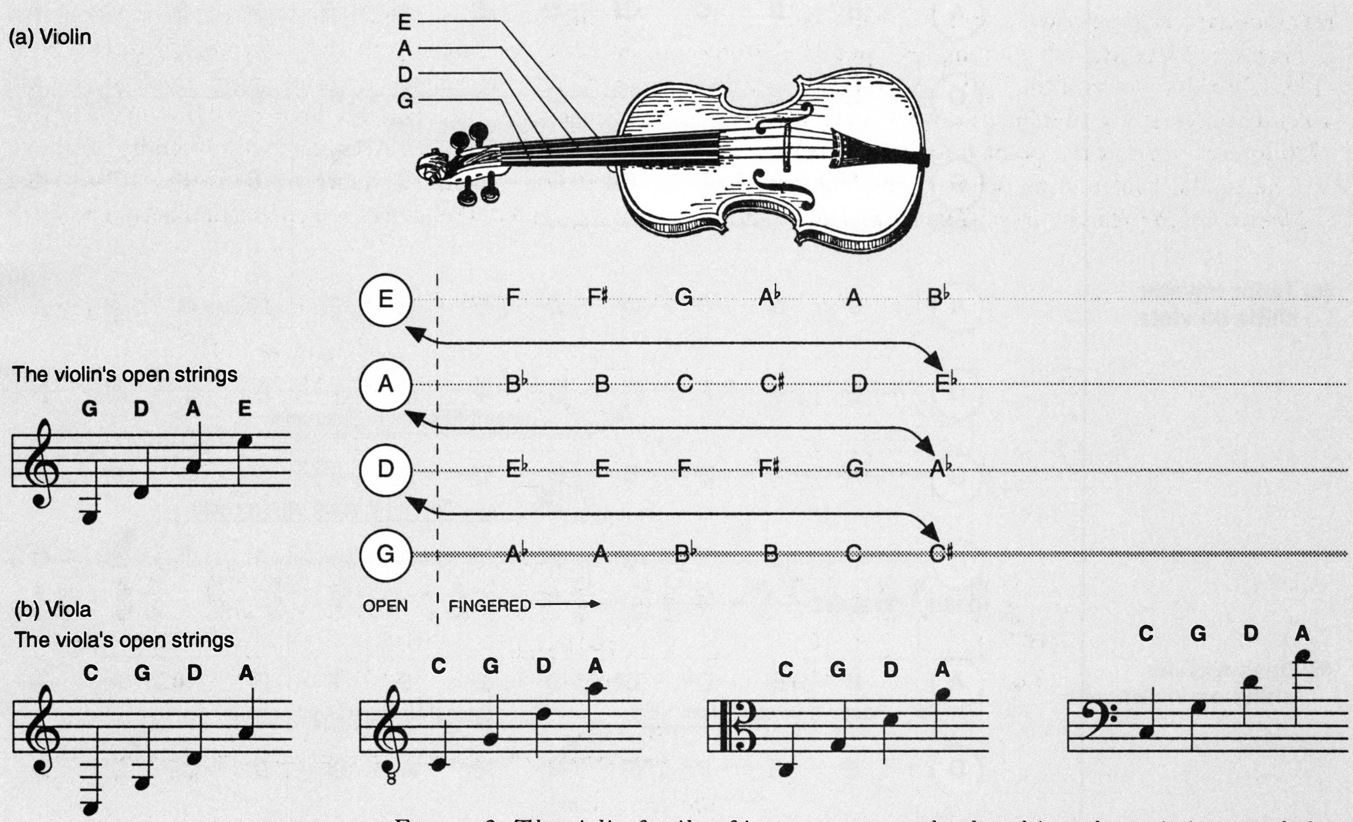
FIGURE 1. *The six species of human singing voice, and their registers.*

discovery and the *Motivführung* revolution, such that mastering the relevant feature of the former leads us to recognize the most crucial feature of the latter.

Classical music is a form of language, derived from the polyphonic vocalization of Classical forms (e.g., Sanskrit) of poetry. To the degree the vocalization follows the physiologically natural pathway of Florentine *bel canto* voice-training, to a well-tempered polyphony centered upon the C = 256 cycles of the child soprano voice, the formal rudiments of the musical language's philology are properly situated for study. The crucial issue then confronts us: "If music is a form of language, to what class of objects does this form of language refer? What is the proper subject of this language called 'music'?"

The subject of Classical polyphony is not the sensuous (e.g., "erotic") features of the musical-language medium (e.g., not "overtones"), but, rather, a different class of object, different than the musical medium as such. To argue to the contrary effect, is as if to propose that the subject of the mathematics professor's classroom oration, is to cause pleasurable sensations in the student's hearing apparatus, or to propose that, for the famished person, the primary object of eating is to amuse the taste-buds.

It is a fair summary, to say that music, like all Classical art-forms, has the necessary object of imparting the combined experience of both *natural* and *artistic* beauty.[13]



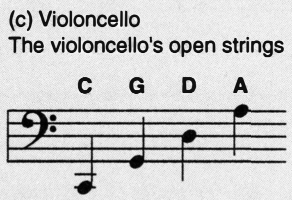


FIGURE 2. *The violin family of instruments was developed in order to imitate, and then extend, the principles of the* bel canto *singing voice. Each member of the violin family has four strings, with each string tuned at the musical interval of a fifth above or below the adjacent string or strings. (a) In the simplest case, each open (unfingered) string of the violin can be used as the lowest tone of a new surrogate "vocal" register. The succeeding higher tones fingered on that string remain in the same "register," until the player changes to the next-higher string. For example, a register shift is simulated by moving from the C♯ played on the G string, up to the open D string—simulating, for instance, the contralto's shift from first to second register. (b) The same principle applies to the open strings of the viola—C, G, D, and A. Because the viola's range straddles the usual treble and bass clefs, for clarity the same four strings are shown here using four different clefs: the treble clef, the modern tenor clef (sounds one octave lower than the treble clef), the alto clef (in which most viola music is written), and the bass clef. (c) The violoncello's open strings.*

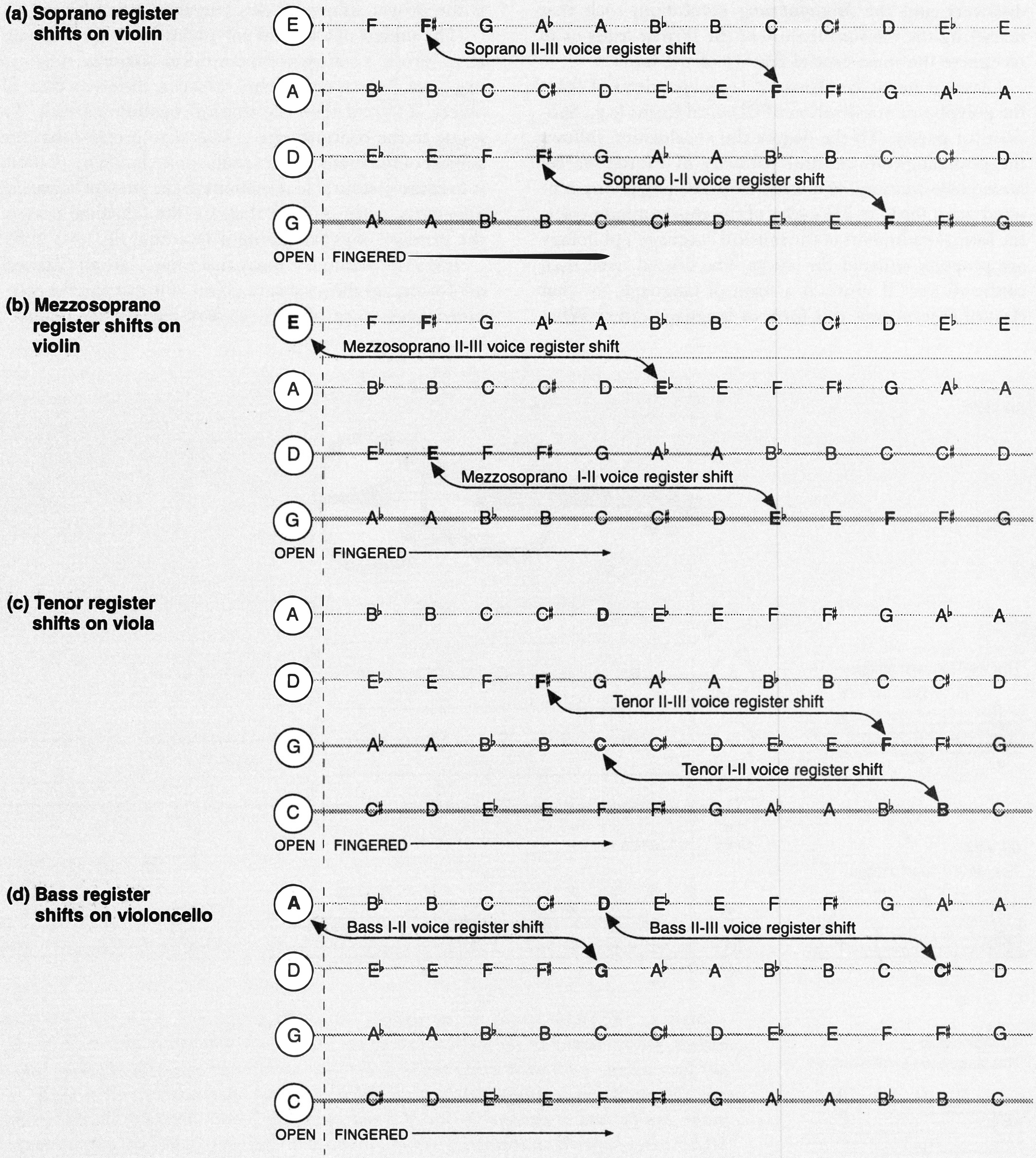


TABLE I. *Each member of the violin family can be fingered in such a way that it can imitate the register shifts of any voice singing within that instrument's range. Here, the violin is shown imitating (a) the soprano, and (b) the mezzosoprano vocal register shifts. For example, the soprano's I-II register shift is imitated by shifting from a fingered F on the G string, to a fingered F♯ on the next-higher D string. The viola is shown imitating the tenor voice species, and the violoncello ('cello) the bass voice. Because these shifts can be made in various places, there are many other possible imitations; also, the four "benchmark" examples shown here are not necessarily the most frequently used. The reader is encouraged to find other possible imitations.*

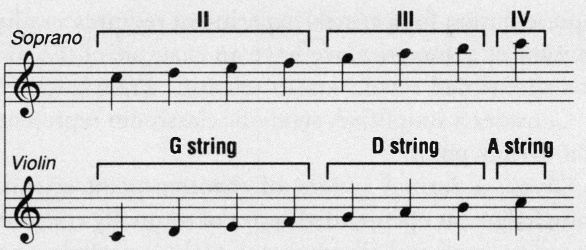
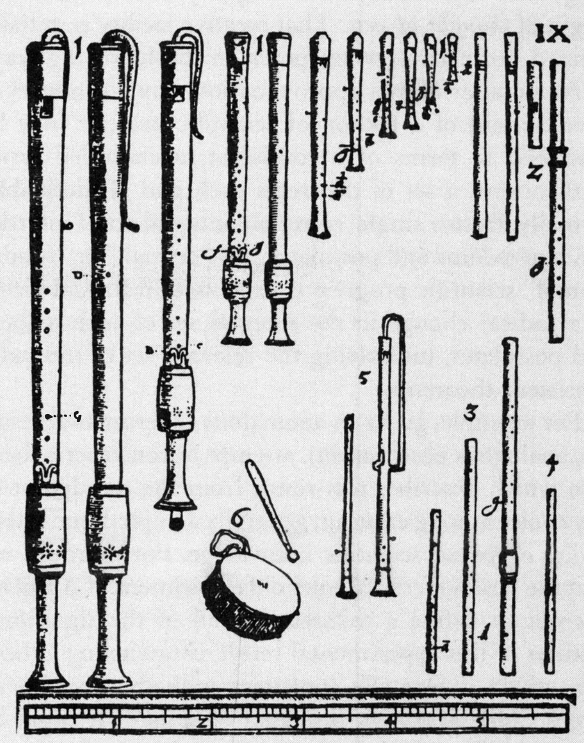


FIGURE 3. *The violin plays the passage sung by the soprano, but at a displacement one octave lower. The octave displacement enables the violinist to imitate the II-III register shift by changing from the G string to the D string, and then the III-IV shift by switching to the A string.*



Source: Michael Praetorious, *Syntagma Musicum*, 1619

FIGURE 4. *By the nature of their construction, the woodwind instruments have registrations which are essentially fixed, even though they can be modified to some degree by choosing alternate fingerings for the same note. The wind instruments therefore tended to be designed and produced in sets or "chests," whose members mostly corresponded to a particular species of singing voice. Above: a woodcut diagram of various wind instruments in use in the early seventeenth century.*

This begs the question: what is the object to which such ideas of beauty correspond? The proper response to the question is Plato's *ideas,*[14] or Gottfried Leibniz's *monads,*[15] or Bernhard Riemann's *Geistesmassen,*[16] or my own choice of term, *thought-objects.*[17] The proper subjects of Classical polyphonic compositions, are *musical thought-objects.*

The essential, deeper psychological features of this *Motivführung* revolution cannot become intelligible, without the following *Type* of direct reference to the subject of the *monad,* or *thought-object.* Since music is a form of language implicit in polyphonic forms of poetic vocalization (according to physiologically natural *bel canto* principles), it, as a medium of communication, must choose a subject for its utterance. It is the essential nature of well-tempered polyphonic development, that the subject of a Classical polyphonic composition *cannot be a symbolic treatment of a sensuous object.* It can be only a different type of object, an object of the intelligence, not the senses; it must be a *thought-object.*

It is therefore necessary to detour briefly from music as such, to set forth summarily some crucial points from the referenced "Metaphor" paper.[18]

II. What Is A 'Thought-Object'?

Humans are the only mortal species of living creatures which is capable of willfully improving, indefinitely, its *potential population-density* (*per capita,* and per square kilometer of average land-area). Those failed cultures so much admired by the anthropologists, are forms of society which, at a certain point, failed to promote ways of life consistent with adequate negentropic rates of scientific and technological progress. Despite the fact that so many cultures have failed in this way, other cultures, which did not fail so, have risen to take the leading place—at least, up to the present time. Thus, despite the fact that so many cultures, in their turn, have failed, the human species as a whole has achieved within its ranks as a whole a net scientific and technological progress, without which civilization would not have survived in any part of this planet.

That faculty, by means of which mankind generates, transmits, and assimilates scientific and technological progress, is the individual person's *divine spark* of potential for rigorous forms of creative reason. This *spark* is the sole basis for the individual person's species-likeness to the Creator; this *spark* is the locus of that quality called *imago viva Dei.* This creative agency, this *spark,* is the origin of *thought-objects.* That creative facility is initially defined, for classroom purposes, in the following way.

As a matter of first approximation, any given level of development of a faction of scientific practice may be described in terms of a consistent, open-ended series of theorems, a set of theorems each and all derivable, formally, from a single, common, integral set of interdependent axioms and postulates. All "crucial," or "fundamental" scientific progress is expressed in formal terms as a radical change in the integral set of such axioms and postulates, underlying the relevant set of mutually consistent theorems.

For example, given an anomalous experimental result (or, analogous observation), attempt to construct a theorem which describes this result from the standpoint of any choice among existing, generally accepted, consistent bodies of *formal* scientific knowledge. For example, repeat the famous, crucial solenoid experiment of Ampère; attempt to define a theorem for all of the significant features of this experimental result, constructing a theorem which is formally consistent with the doctrine of James Clerk Maxwell; it cannot be done![19] It could be done only if a radical change is imposed upon the axiomatic assumptions commonly underlying the dogmas of Clausius, Kelvin, Helmholtz, Grassmann, and Maxwell.[20] In such as the latter case, in which a fair theorem representation for a crucial experiment requires a radical revision of axiomatics, we have an example of the form of a threatened revolution in scientific knowledge.

Consider a simplified, symbolic classroom representation of this point.[21]

Given, a formal system of theorem-point scientific knowledge: an open-ended series of mutually consistent theorems, each and all consistent with an underlying set of intradependent axioms and postulates. Call this a "theorem-lattice." Begin with such a theorem-lattice, *A.* Introduce a crucial, real-life experiment, X_1, for whose result no theorem may be constructed which is consistent with *A.*

Now, there exists at least one radical revision of *A*'s underlying set of axioms and postulates, which permits the construction of a formally consistent theorem for X_1; there may exist many such revisions which satisfy this bare condition. However, we must satisfy not only the evidence of X_1; we must also satisfy every crucial experiment which corresponds to the subject of any other theorem of *A.* This restricts the choices of radical revision for *A*'s axiom-set. In the case this condition is met, we have a new theorem-lattice, *B.*

Thus, in similar fashion, define a series of mutually inconsistent theorem-lattices, *A, B, C, D, E, ...* Since

Instrumental Imitation of the Singing Voice

In this passage from Mozart's Mass in C, K. 427, the solo soprano voice introduces a phrase which serves as a transition back to the concluding full choral section. The solo soprano is accompanied at the unison by the Violin I, while the Violin II plays a pedal-point B♭. Then the chorus enters during the fifth measure of this example; the choral sopranos sing the same line as the solo soprano before, but the Violin I now plays the line at a displaced range, one octave higher. The Violin II now plays with the sopranos at the unison, and the oboes take over the B♭ pedal point, one and two octaves higher than the previous Violin II pedal point.

each theorem-lattice is separated from its predecessor by a radical change in the implicitly underlying set of interdependent axioms and postulates, no two lattices are consistent, and no theorem of one lattice is consistent with any theorem of any other lattice. This is a higher expression of what is termed a "mathematical discontinuity"; in this case, a formally unbridgeable chasm separating each term of the series from every other term of the series.

In the real universe, as reality may be distinguished from mere formalities, the test of the validity of the series, *A, B, C, D, E, . . .* , is posed by the question, whether the successive changes in modes of society's productive (and, related) behavior, effects resulting from employment of changes in scientific knowledge, do, or do not represent implicitly an increase of the rate of growth of society's potential population-density. In the case that this test is satisfied, the series as a whole represents (and is represented by) *a subsuming method* of generating revolutionary successions of advance in scientific and technological progress.

The advances in productivity (and, potential population-density) which European culture has achieved (over the anti-growth oppositions), during the past 550 years, since the 1439-1440 A.D. Council of Florence, are implicitly the outgrowth of radical axiomatic changes in creative scientific thinking. These changes can be represented most efficiently, most intelligibly, from the standpoint of a non-algebraic function's reference-point in a radically constructive synthetic geometry. This history, seen through the eyes of such a non-algebraic geometry, permits the easiest rigorous method for introducing the meaning of *thought-object,* whether for physical science, or for music.

This modern history's most elementary, pivotal discoveries can be reduced to a short list.[22] From ancient Classical Greece (including southern Italy), two geometrical discoveries are outstanding: the famous Pythagorean Theorem, and Plato's extensive treatments of those five regular polyhedra which may be inscribed within a sphere (the "Platonic Solids").[23] The method associated with these discoveries, is the Socratic dialectic, as typified by Plato's *Parmenides* dialogue, a method which Plato stressed as congruent with a radically constructive synthetic geometry.[24] The rise of modern science, resting upon the Greek heritage of Pythagoras, Plato, and Archimedes, begins with the discoveries of Cardinal Nicolaus of Cusa and his collaborators, about 550 years ago, centered around Cusa's *De Docta Ignorantia* (On Learned Ignorance).[25]

The most crucial discoveries in modern physical science occurred during an interval of approximately 250 years, from c.1440 A.D. through the beginning of the eighteenth century. The 1696-1697 A.D. solution to the brachistochrone problem, by Leibniz and the Bernoullis, is typical of the flood of final touches on the first quarter millennium of modern scientific progress.[26] From this period, the following are the most notable. (1) Cusa's 1430's discovery of the "isoperimetric" ("Maximum-Minimum") principle, the root of the later principle of non-algebraic "least action."[27] (2) The further elaboration, by Leonardo da Vinci and his collaborators, of the implications of the "Platonic Solids."[28] (3) The establishing of the first comprehensive program in mathematical physics, by Johannes Kepler, principally upon the basis provided by Cusa and Leonardo.[29] (4) The seventeenth-century development of a Keplerian, non-algebraic calculus of physical "least action," by Pierre Fermat,[30] Blaise Pascal,[31] Christiaan Huygens,[32] Gottfried Leibniz, and the Bernoullis.[33] It was in this Renaissance setting of vigorous scientific progress, that the rise of Classical polyphony through Leonardo da Vinci,[34] Bach, Haydn, Mozart, Beethoven, Brahms, *et al.* occurred.

At first inspection, geometric discoveries are apparently, merely mathematical formalities, in the sense algebra is in fact merely empty formalism. We have already indicated here, that the validity of a succession of formal revolutionary discoveries is tested by the yardstick of potential population-density. For obvious reasons, physics, chemistry, and biology, combined as one, insofar as they reflect man's increase in power over nature—*per capita,* and per square kilometer—are an implied approximation of increase of potential population-density. Since the middle of the fifteenth century, the development in empirical authority of non-algebraic mathematical science has been premised upon the universal principle of physical least action: least action in physical space-time, a concept rooted in Cusa's isoperimetric, "non-algebraic" circle, the least (circular) perimetric displacement subsuming the relatively largest area. Throughout that 250 years or so, this principle of (physical) least action has been situated in respect to two interdependent physical phenomena: electromagnetic radiation and hydrodynamics. Even today, all sound experimental physics relies upon those non-algebraic species of formal functions which locate physical reality in terms of the hydrodynamics of electromagnetic least action.

It is in that setting, of geometrical and physical thought, combined, that the easiest definition of a thought-object may be supplied. From that vantage-point, in turn, the nature of a musical thought-object follows readily. Resume the elaboration of the theorem-lattice series.

Given, the indicated series of theorem-lattices, *A, B, C, D, E, ...* Define a function which subsumes the generation of the successive terms of this series. Since no two terms of the series may be consistent, no formal function for the series can be defined by means of the terms denoting specific theorem-lattices. Rather, even by mere definition, the generation of *B* from *A*, *C* from *B*, and so on, lies in that which generates the *absolute* quality of formal discontinuity between any two terms of this series. That generation is the *radical change* in axiomatics, so altering the implicitly underlying set of interdependent axioms and postulates.

There is a "mapping correspondence" between this agency of radical change and the discontinuities separating the terms of the series. Those radical changes correspond to thought-objects. That is what we must define, before returning to the musical thought-objects.

There are two distinct species of thought-objects implied in the given, illustrative series of theorem-lattices. First, on the relatively lower level, there is a quality of the thought-object which is typified by the transformation of *A* to generate *B*. Second, there is the higher quality, higher species of thought-object associated with a notion of a choice of determined ordering for the series presented, the ordering of the lower-order thought-objects corresponding to the discontinuities $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, ...

For example, a successfully advancing science would be associated with a succession of such revolutions, each always leading the relevant society (implicitly) to higher levels of potential population-density. This would also signify, that that generation of successive revolutions $\overline{AB}$ and $\overline{BC}$ must result in a revolution $\overline{CD}$, which latter increases the *potential* population-density more rapidly than the average of $\overline{AB}$ and $\overline{BC}$. These successive revolutions are effected under the guidance of a self-evolving method for effecting successive such revolutions, a self-evolving method of scientific discovery. Call this quality of revolutionary ordering a method of *evolutionary negentropy* in increase of potential population-density.

Understand "evolutionary negentropy" as a conception introduced by Nicolaus of Cusa.[35] The progressive evolution of the biosphere is dominated by emergence of relatively higher species—higher than any previously extant. This does not (generally) wipe out the surpassed inferior species. Rather, the proliferation of most among the accumulated, interacting species makes possible the emergent existence of the higher species. Similarly, in the case of the Mendeleyev Periodic Table of Elements and their Isotopes, the emergence of helium and lithium, and so on, from nuclear fusion of hydrogen, and so on, does not eliminate the lower ranking elements and isotopes of that table; rather, that development is characteristic of an ever higher state of organization of the "table" as an interdependent wholeness.

We combine this view of such revolutionary/evolutionary processes as these, with a notion of rising "free energy" of the entire "system" undergoing such ordered evolution. This combination of higher states of organization with relative increase of "free energy," is a definition we prescribe for our use of the term "negentropy."

Thus, we have our two species of thought-objects, relative to our illustrative series of formal theorem-lattices. The first, relatively lower species, is associated with the *Type*[36] of discontinuities separating *A* from *B*, and so on. The second species, a cousin of the *Motivführung* principle, is associated with the *relative evolutionary negentropy* of the whole series as a *determined* series as a whole.

There exists no medium of communication within whose terms either species of thought-object might be represented *explicitly*. No form of algebra, nor of other species of formal language-medium, could represent such a thought-object *explicitly*. Thought-objects belong to a class of distinct mental existences which have no functional correspondence, or equivalence to those representable sensory images which are the type of explicit objects of formal communication.

The same is true, of course, of musical thought-objects, such as the thought-objects corresponding to any among the three principal discoveries upon which the *Motivführung* revolution depends. This is to emphasize, that that creative faculty, the means by which Leonardo da Vinci effected his fundamental scientific discoveries was the same higher, ("negentropic") *methodological* thought-object which directed his principal compositions in music[37] and plastic arts. Notably, in the plastic arts, Leonardo's medium of discovery was that same set of geometrical principles governing his fundamental discoveries in physical science.

Yet, in both aspects of Leonardo's creative output, no mere symbolic device could represent the relevant thought-object. Nonetheless, there do exist indirect means for communication of thought-objects, with certainty, from one mind to another. Ironically—"ironical" in a most meaningful dual sense—these indirect means, known as Plato's "Socratic," or "dialectical" method, are more efficient agencies for communication than any formal medium could become. Not only is the Socratic dialectic more efficient than the banal, nominalist Aristotelian formalism; the Socratic dialectic efficiently imparts those classes of conceptions which are so powerful, so profound, that the gnostic Aristotelians, such as Immanuel Kant, avow these conceptions to be intrinsically "unknowable."[38] These thought-objects are otherwise known as "Platonic ideas."[39]

Classical music demands a method of polyphonic composition equivalent to that Socratic dialectic. This method, applied to that developed form of the musical medium, is employed to the effect of imparting, indirectly, a sub-class of otherwise "unutterable Platonic ideas," called usefully either "musical ideas," or, with less ambiguity, "musical thought-objects."

The point has been reached, here, to identify the class of phenomena of inner mental experience which contain the marks of the thought-object.

III. The Principle of Least Action

Let us resume here with a partial restatement of what has been said thus far. The crucial feature of the Christian "Golden Renaissance's" launching of modern science, approximately 550 years ago, is Nicolaus of Cusa's discovery of his isoperimetric ("Maximum-Minimum") principle.[40] As this Renaissance picked up from the point at which Classical Greek civilization had been interrupted, that by the evil, Gaia-Python-Dionysos-Apollo Cult of Delphi,[41] so, Cusa began the modern scientific revolution at approximately the point Archimedes' work was snuffed out by the brutish legionnaires of Delphi's pagan Rome[42]: Archimedes' paradoxical theorems on the subject of "squaring the circle."[43] This crucial discovery by Cusa is aptly described, alternately, as a unique physical principle of "least action"; so, it appears more clearly in retrospect, by the close of the seventeenth century. This comparison of two discoveries, presented in 1440 and 1697, respectively, serves us here as our exemplary choice of model for a thought-object.

On closer, stricter scrutiny, the term "squaring the circle" is ambiguous. Its cruder meaning is, simply: to construct a square whose area is nearly equal to that of a given circle. This task was solved, implicitly, by Archimedes and others.[44] There is, however, a subtler feature. This subtler task is, to construct the perimeter of a circle by linear, or "algebraic" methods; this second, subtler task is an impossible one, for reasons shown conclusively in a solution constructed by Nicolaus of Cusa. That latter solution is the point of reference for our constructive, indirect, but rigorous definition of a thought-object.

These various points are each and all clarified by closer scrutiny of Archimedes' four theorems on the squaring of the circle; this is the approach employed successfully by Cusa.[45] We now describe this summarily.[46]

Inscribe a square within a circle. Circumscribe that circle with a second square (SEE Figure 5). Double the number of sides of each square to form a pair of a regular octagons in the same relationship to the circle as the pair of squares. Repeat the doubling action, to reach a large value of 2^n sides. Look at the region of the circle's perimeter associated with three or four sides of an inscribed polygon of very many sides (SEE Figure 6). By estimating the area of both the inscribed and circumscribed polygons, respectively, and by averaging the two areas, we have a rough estimate for the area of the circle; however, the perimeter of neither polygon could ever become congruent with the perimeter of the circle.

Let the diameter of a given circle be one meter. Dividing the estimated perimeter of the circle by one meter, gives us an estimated value for π. However, respecting

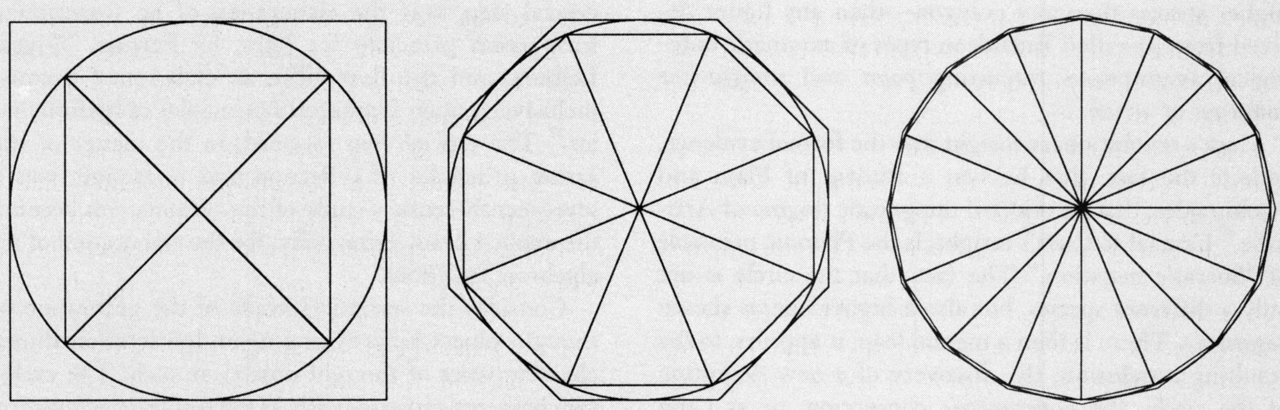


FIGURE 5. *"Squaring the circle": Estimating the area of a square approximately equal to that of a given circle, as the average area of two regular polygons.*

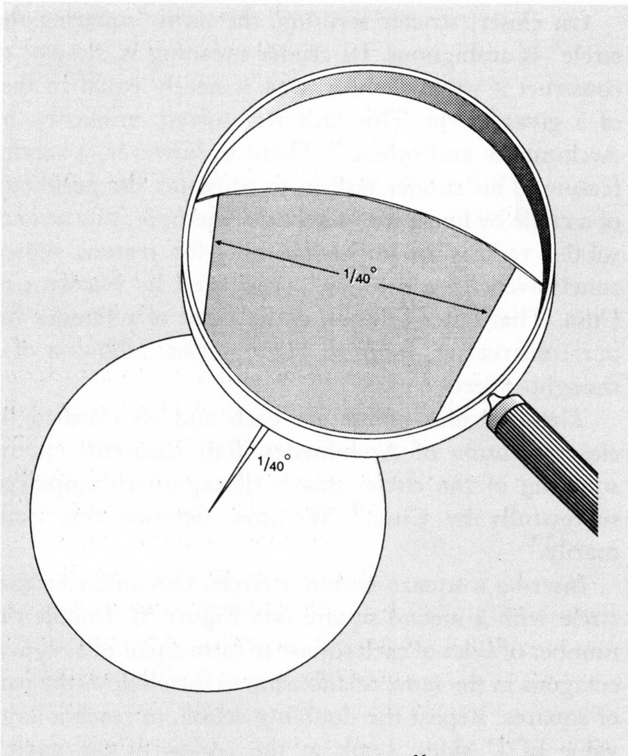


FIGURE 6. *An inscribed polygon of 2^{16} (65,536) sides may seem to closely approach a circle. But the perimeter of the polygon can never become congruent with the circle's perimeter.*

either polygon, even if we increase the number of sides of an 2^n-sided regular polygon to the astronomical $n = 256$, there would remain a well-defined, distinct, functionally determined discrepancy in area between the polygon and the circle. Worse, the many-angular perimeter of the polygon becomes ever less congruent in *species-form* with a circular perimeter. The circle belongs to a different, higher species than any polygon—than any figure derived from so-called Euclidean types of axiomatic ontological assumptions respecting *point* and *straight-line pathways of action.*

Cusa's revolutionary insight into the formal evidence, reflects the fact, that he was a student of Plato and Archimedes, that he rejected the gnostic dogma of Aristotle.[47] Crucial to Cusa's insight, is the Platonic principle of "Socratic negation." The fact, that the circle is not only a different species, but also a higher one, is shown *negatively.* There is then a mental leap, it appears, to the resulting conclusion: the discovery of a new definition of the circle, the *isoperimetric* conception, or as Cusa defines it, his "Maximum-Minimum" principle.[48] However, appearances aside, this discovery is no "blind leap of faith"; Cusa was already a master of Plato's Socratic method; he was familiar with "Platonic ideas."

The remainder of the ensuing two-and-one-half centuries of fundamental scientific progress, was an elaboration of Cusa's isoperimetric principle as the emerging, universal principle of physical least action. Some preliminary observations on this connection are needed, to clear the way for our next major point.

During the nineteenth century, the famous Professor Jacob Steiner, the author of the synthetic geometry curriculum for quality secondary schools,[49] contributed a standard classroom demonstration of the iterative, isoperimetric construction of a circle. Although the Steiner construction helps, it must be used as a kind of *negative* demonstration, and not positive determination of the circle as a species. There is no formal way in which the isoperimetric circle might be generated positively from the standpoint of a Euclidean theorem-lattice.[50] The notion of the isoperimetric circle becomes "as if" self-evident, replacing thus axiomatically the no longer self-evident, merely derivative point and straight line. Steiner's construction *does not prove* Cusa's isoperimetric principle; it illustrates the result negatively, and this from the standpoint of a good quality of secondary-school classroom. After Cusa, the greatest, most fruitful scientific thinkers, beginning with Leonardo da Vinci, treated the circle (and the sphere) as species which exist "self-evidently," and treated other forms as existences which must be derived, by construction, from the point of origin of circular (and spherical) isoperimetric action (in physical space-time). This work focused upon the anomalies of perspective and vision from the vantage-point of origin of isoperimetric, or "least action."

The first next major step for science, was exploring the implications of the "Platonic Solids." This resulted in such crucial accomplishments as the Leonardo-Kepler functional distinction between the two curvatures (positive and negative) of the circle and sphere.[51] The next crucial step, was the elaboration of an isoperimetric, least-action principle for light, by Fermat, Huygens, Leibniz, and the Bernoullis, an elaboration premised, inclusively, upon Leonardo's principles of hydrodynamics.[52] The crucial step forward, in the matter of least-action principles of reflection and refraction, was the seventeenth-century study of the cycloids, this becoming the explicit basis, principally, for the elaboration of non-algebraic functions.

Consider the second example of the generation of a thought-object, before bringing under closer scrutiny the characteristics of thought-objects as such. The cycloids are characterized essentially as the results of *axiomatically circular action upon axiomatically circular action.* These represent the original, primary form of developable function in the physical domain; they serve, thus, as the axiomatic basis for synthetic-geometrical representation of physical processes as phenomena. This circular action is deemed axiomatic, so, replacing in this way the now merely derived existences of point and straight line. The relatively most elementary *ontological* results of such circular action upon circular action, are twofold: first, least-action function as a characteristic of all action in physical space-time (SEE Figure 7); and, second, an affirmation of Kepler's distinction between functions determined, respectively, by negative and positive spherical curvatures (SEE Figure 8).[53] Situate Bernoulli's 1697 treatment of the least-action equivalence of the *brachistochrone* to Huygens' *tautochrone,* in this context (SEE Figure 9).[54]

The result, the proof that radiation of light occurs in a universe which is curved relativistically, in physical space-time premised elementarily upon uniquely axiomatic least action, is a thought-object solution developed, in the late seventeenth century, as if by a leap of faith, from a process of Socratic negative reasoning driven rigorously to its limits.

In each of the listed cases of discovery, three general results dominate. Firstly, each, Cusa's, Leonardo's, Kepler's, Huygens', Leibniz's, and the Bernoullis', is generated by the same type of apparent "leap of faith," under analogous circumstances. These circumstances are a paradox driven toward its limit, by means of an exhaustively rigorous application of Socratic dialectical negation, a negation analogous to the method of Plato's *Parmenides.* Secondly, excepting Cusa, who depends upon ancient crucial discoveries, none of the other discoveries listed had been possible without all of its predecessors in that same series. Thirdly, each discovery, and all combined the more so, increased greatly mankind's power over nature, mankind's potential population-density.

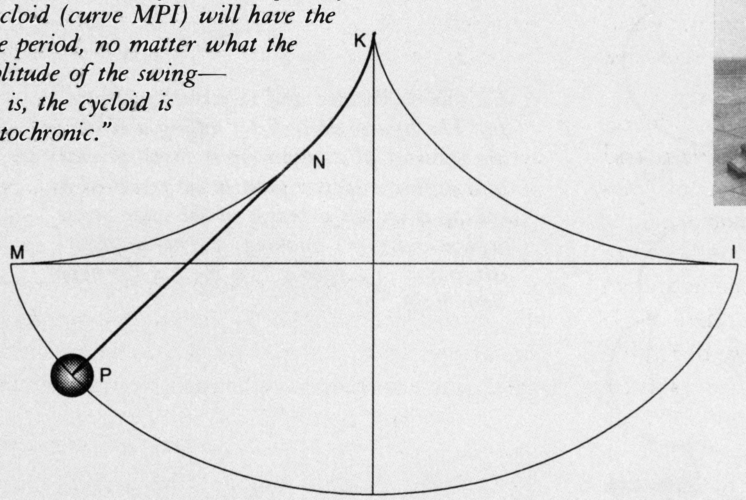
The 1890's work of Georg Cantor,[55] David Hilbert's formalist error on proposing his famous "Tenth Problem,"[56] and the case of Kurt Gödel's famous proof, all illustrate deeper implications of our deceptively simple series of theorem-lattices, *A, B, C, D, E,* . . . .

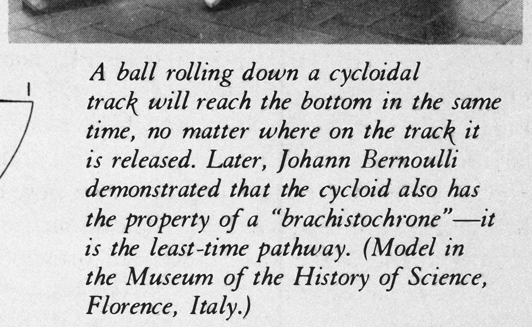
Let us substitute for the commas in that series, the letter μ, to such effect that we have, in first approximation, the new representation of that series, μ_{ab}, μ_{bc}, μ_{cd}, ... Each of the terms now appears to correspond to a successful "leap of faith," to Kant's purportedly "unknowable" agency of creative discovery. This cannot yet be an adequate representation; two general grounds of that warning are to be indicated. Firstly, without the discoverer's earlier reproduction of numerous similar "leaps of faith" of his predecessors, his own "leap of faith" were impossible, rather than successful, as it was. Secondly, this functional (e.g., *analysis situs*) ordering of the formal series correlates with a twofold increase of mankind's potential *per-capita* power over nature: on account of the individual discovery, as such, and, also, additionally, on account of the contribution to the increased power for discovery by society in general.

Shift our view, momentarily, to the Classical humanist classrooms of Europe, from the Grootean teaching order, the Brothers of the Common Life, through the German *Gymnasium* of Wilhelm von Humboldt's design. The relevant feature of that classroom, is emphasis upon use of primary sources' representation of processes of great

FIGURE 7. *The least-action principle embedded in cycloid functions.*

In his 1673 On the Pendulum Clock, *Huygens demonstrated that a pendulum made to follow the path of a cycloid (curve MPI) will have the same period, no matter what the amplitude of the swing—that is, the cycloid is "tautochronic."*





A ball rolling down a cycloidal track will reach the bottom in the same time, no matter where on the track it is released. Later, Johann Bernoulli demonstrated that the cycloid also has the property of a "brachistochrone"—it is the least-time pathway. (Model in the Museum of the History of Science, Florence, Italy.)

discovery, prompting the student, in this way, to replicate that mental experience of the discoverer in the student's own mental processes.

The act of discovery is not represented explicitly in any primary source. That action is not explicitly representable in any medium of communication. Nonetheless, a fair replica of the original act of discovery may be evoked from within the creative potential of the student's mental processes. In that degree, that aspect of the creative intellects of Pythagoras, Plato, Archimedes, Cusa, Kepler, and so on, lives anew as an integral capability of the mind of the student. So, it may be said fairly, the noble dead may communicate, by such dialectical indirection, as if directly, mind to mind, with the living. Such is true education, unlike that sterile textbook drill and grill, which rehearses today's pupils to pass computer-scoreable multiple-choice questionnaires. Thus, by the methods of Christian humanist education, the quality of true "genius" is learned, by incorporating in one's own creative-mental processes a choice selection of bits of the mental processes of a large number of the greatest discoverers, such as Plato, of mankind's past.

Consider the exemplary case of one of the greatest thinkers in all recorded history, Nicolaus of Cusa. His education was shaped by the influence of that great Grootean teaching order, the Brothers of the Common Life. He assimilated thus, for example, the minds of Plato and Archimedes, and many others. Or the illustrious case of Leibniz's collaborator, Christiaan Huygens.[57] Christiaan's father, Constantine, was a celebrated Dutch diplomat, a co-sponsor of the young Rembrandt van Rijn, and one-time ambassador to London. In London, father and son Huygens gained access to the Royal Collection of Leonardo da Vinci's papers, whose contents played later a direct part in important work of both Christiaan Huygens and Leibniz.[58] The work of Cusa was known to these circles; the work of Kepler dominated the seventeenth century, and was later, the foundation for much

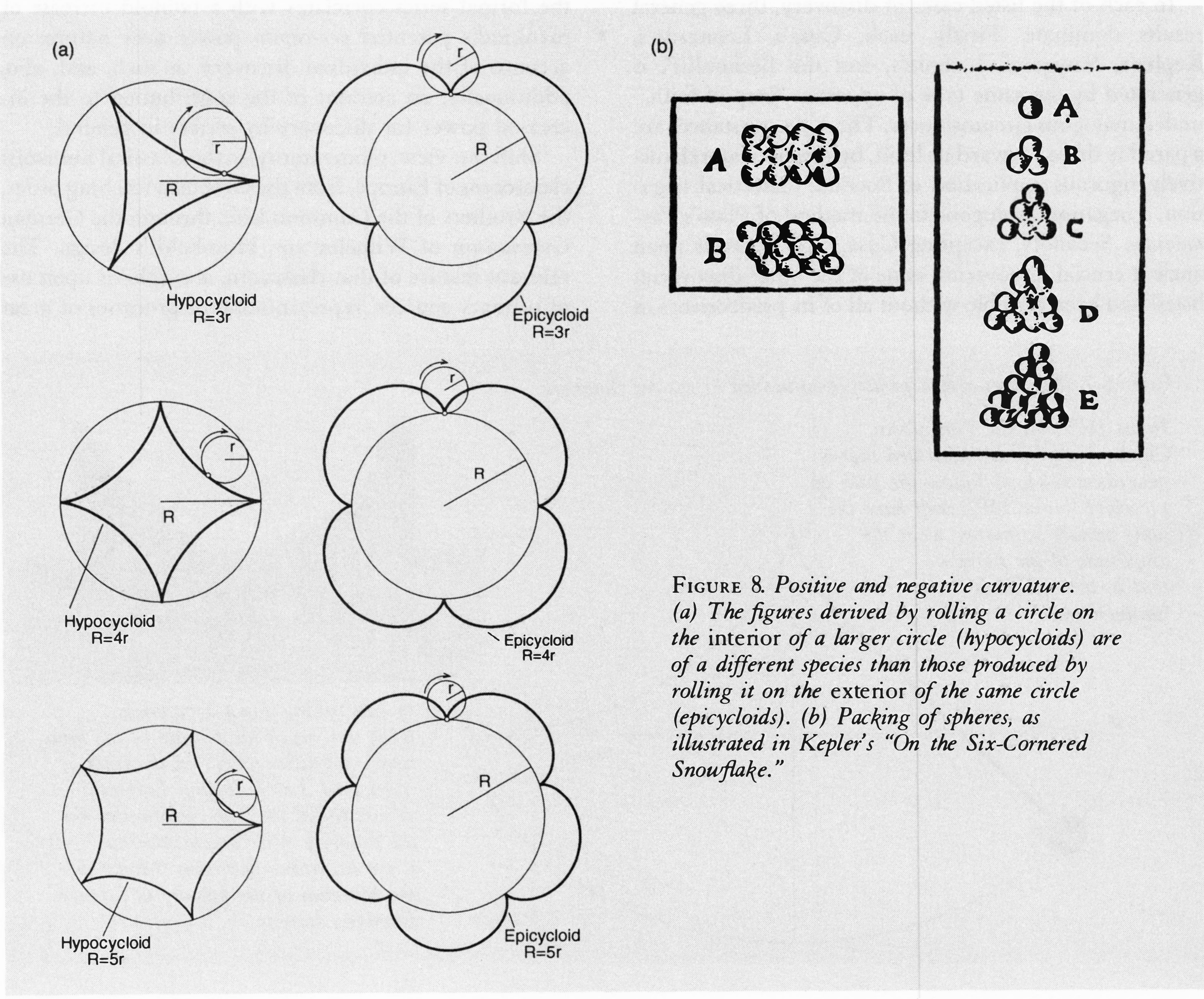


Figure 8. *Positive and negative curvature. (a) The figures derived by rolling a circle on the* interior *of a larger circle (hypocycloids) are of a different species than those produced by rolling it on the* exterior *of the same circle (epicycloids). (b) Packing of spheres, as illustrated in Kepler's "On the Six-Cornered Snowflake."*

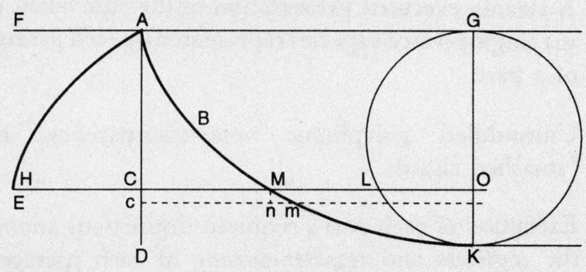


FIGURE 9. *In 1697, Johann Bernoulli demonstrated that cycloid AMK had both the "tautochronic" property shown by Huygens, and the property of a "brachistochrone"—that is, it is the least-action, least-time pathway of descent.*

of the work of Carl Friedrich Gauss. Leibniz's founding of the first successful differential calculus, circa 1676, in Paris, France,[59] was, like other attempts of that period, prompted explicitly by Kepler, and contained the work toward that end in Leibniz's study of Blaise Pascal's unpublished notes, as well as Pascal's published work. Consider, to similar effect, two Platonic dialogues composed by Leibniz for the stated purpose of demonstrating that crucial issues of science today require resort to Plato's dialectical method.

Science is not the sterile pedagogue's obsession with statistical procedures for "inductive" generalization from a caddis-fly pupa's aggregation of so-called "facts" and recipes. Science is, historically, the development and interaction of those higher species of mental life which are here designated as the thought-objects, generated by creative activity, which Leibniz termed *monads.*

All of us who have effected successfully some discovery of a natural principle, as this reviewer did, decades ago in his contributions to the science of physical economy,[60] know that thought-objects are fully intelligible, although not susceptible of an explicit, sensory form of representation in any formal medium of communication. We also know that our successful work is modeled, as if "heuristically," upon our learning experience in reproducing within our own creative mental processes the thought-objects corresponding to valid acts of discovery of principle by as many as possible among all the greatest thinkers before our time.

Thus, the provisional array of such thought-objects, μ_{ab}, μ_{bc}, μ_{cd}, . . . , is subsumed by a generative, self-evolving quality of yet higher-order thought-object. This higher species of such thought-object is called *scientific method,* a thought-object whose efficient dimensionalities are the notion of "evolutionary negentropy," which we referenced above.

IV. Musical Thought-Objects

In its most essential features, what we may say of thought-objects, as in scientific work, we may say also of musical thought-objects. The J.S. Bach *Musical Offering* underscores the place of a major/minor-key cross-over *dissonance*—e.g., a formal discontinuity—in the process of composition. The subsumption of many resolved discontinuities under the governance of a single, well-defined ordering-principle for that succession as a whole, presents us, in the instance of any single such composition, with a process analogous to the idealized theorem-lattice, *A, B, C, D, E,* . . . .

As long as the composer adheres strictly to the natural lawfulness of *Classical* well-tempered, *bel canto*-rooted instrumental, and vocal polyphony as polyphony, certain dissonances, such as the F♯ of those Classical C-major/C-minor *Motivführung* compositions quoting from Bach's *Musical Offering,* are defined meaningfully as formal discontinuities, to be resolved as such. (In strict Romanticism, or atonalism, such rationality is more or less irrelevant.) Thus, the composer's *Motivführung* solution to the *negation* so represented thematically (as in quotations from the *Musical Offering,* by Mozart, Beethoven, Schubert, Chopin, *et al.*), generates a species of *musically-defined* thought-object, or, briefly, a musical thought-object.

The definitional significance of such a musical thought-object as musical, rather than simply a thought-object, is the following. Firstly, even the individual thought-objects, *of a series,* within a succession, are provoked, in the individual's sovereign creative mental processes, by the polyphonic lawfulness of the Classical, well-tempered musical medium. Secondly, the ordering of a series of such thought-objects, as a composition, or part of it, is a higher-order thought-object, which latter is defined, generated by a negative feature of a process of composition. The natural rules of polyphony flowing from singing voices of the most natural training (i.e., *bel canto*) are the basis for defining an anomaly, and, thus, are the basis for the generation of a musical thought-object. In other words, the thought-object is referenced in respect to its place in the development occurring in the musical medium. Since only the Classical mode of composition permits this determination, those musical thought-objects are defined in respect to the Classical form of the medium.

"Show me your thought-object, by indicating to me how this [musical] passage should be performed," would be the way a trained Classical performer would tend to reference the matter being addressed in the foregoing paragraphs. Commonly, among such professionals, it is the shadow of the thought-object, so to speak, which is referenced, not the thought-object as such. The formal heading under which this reference is made, would be, most frequently, "musical insight," a quality whose exact communication may be suggested by apt description, but whose conception is recognized by performance of a relevant musical passage or composition. We may refer to the passage, or the composition as a whole, and speak of a performance-demonstrated insight into the *intent* of that passage; we speak of this as musical "insight."

The pleasure of such musical ideas—musical thought-objects—is akin to that of solving a scientific problem: it is the quality of emotion we associate with "sacred love" (*agapē, caritas*), as distinct from sensuous, object-fixed "profane love." In that respect, all Classical polyphony, all Classical musical ideas (thought-objects), as opposed to the erotic fantasies of Wagner's and Mahler's "Romanticism," have an intrinsic quality akin to the religious feeling of the Gospel of St. John and St. Paul's I Corinthians 13.

In each instance of the series of fundamental scientific discoveries referenced, the most rigorous principles of geometric construction, driving a paradox to its limit, was required—as in Plato's *Parmenides*—to show the ontologically *axiomatic* issue upon whose resolution the matter hangs. In this setting, and only such, is a valid thought-object generated by the individual mind's sovereignly creative agency. In music, similarly, a strictly lawful polyphony, itself rooted in strictly well-tempered, (Florentine)[61] *bel canto* vocalization, is the "constructive geometry of hearing," by means of which the relevant axiomatic issues are posed to the creative agency.

This requirement's character is illustrated by the following exemplary problems of musical performance. There are several, broadly mandatory features of a competent Classical performance, for lack of which rigor the necessary, *indirect* communication of the composer's intended musical thought-object will be impaired, or even may not occur (it should be noted that this does not apply to the performance of Romantic, or atonal compositions, whose subjects are not thought-objects, but rather the smarmy, erotic objects of the Rousseauvian degenerate's program-notes). For a serious Classical composition, such as those of Bach, Haydn, Mozart, Beethoven, Schubert, Mendelssohn, Chopin, Schumann, or Brahms, the conveying of musical thought-objects demands:

1. A *bel canto*-singing quality of both vocal and instrumental parts (a pro-vibrato quality).
2. A cleanly executed presentation of the equivalent of the singing-voice's species represented by each passage of a part.
3. Unmuddled polyphonic voice-transparency: no "smashed chords."
4. Execution of each part's required distinctions among the registers and register-passing of each passage's singing-voice equivalence.
5. A clean, beautiful shaping of phrasing, and of execution of individual tones.
6. No camouflaging of a performer's want of musical insight, as by means of today's increasing occurrence of and recklessness in use of manneristically exaggerated tunings, tempi, and rubati.

The relevance of this list of precautions to the subject of musical thought-objects, not a desire to enter into the subtleties of the performer's master class, obliges us to consider here a few, bare minima which illustrate the preconditions of bare polyphonic literacy of performance needed to render an intelligible insight into the composer's musical thought-objects.

Some commonplace abuses of the modern keyboard instrument illustrate most aptly the varieties of anti-musical "instrumentalism" fostered even in the practice of numerous known performers. A Classical pianoforte (or, fortepiano) work—such as a keyboard sonata of Mozart, Beethoven, or Schubert—does not know of the existence of chords *per se;* it knows chords only as fleeting shadows of an instrumental parody of *bel canto* vocal polyphony. Each tone of such a chord corresponds to a line of a surrogate for some species of singing-voice vocalization. The performer must bring forth that singing quality, shaping the phrasing and individual tone according to appropriate indications of relative register and register-passing.

An excellent choice of illustration of this point, respecting Classical keyboard compositions, is found in the concluding coda of Beethoven's Opus 111. This is one of Beethoven's major quotations of the Mozart K. 475/457 *Motivführung* derivation from Bach's *Musical Offering.*[62] The pianist should perform this coda in his or her mind as a choral work, and then as a string quartet's parody of that choral performance; then, parody that string quartet's performance at the keyboard. Use the reference to the *bel canto* chorus, to define the properly implied singing-voice species, and with the corresponding registration and register-passing. Then bring these ironies forth from the keyboard, with full contrapuntal

FIGURE 10. *(a)The piano score of the concluding coda of Beethoven's Sonata Op. 111 should be read by the performer not as "instrumental piano music," but as a condensed shorthand version of a string quartet score, which in turn is a reflection of an implied "vocal" score. (b) The same passage has been "exploded" into such a four-part "choral" score, with each voice occupying its own staff. The pianist must always be at pains to observe the implicit register changes as shown such a "vocal" score. (For an explanation of the boxed register markings, see footnote 62.)*

transparency (SEE Figure 10).

Next, to the same purpose, let that pianist turn to a related work, the first movement of Chopin's "Funeral March" sonata. This is to be read, of course, as a quotation of Beethoven's Opus 111 (SEE Figure 11). Chopin is a classical composer, not a Lisztian Romantic. His works must be performed with a corresponding polyphonic transparency, without mannerism, not brutishly slaughtered as if in some pagan's human sacrifice, upon the altar of eroticism.

To the same purpose, turn to a selection from Mozart's post-1781 compositions. Include at least, his 1785/1784 C-minor Fantasy-Sonata K. 475/457, and his C-minor 1788/1783 Adagio and Fugue K. 546/426. Perform—in the mind, as well—first, the two-keyboard K. 426, performing it as if it were a keyboard echo of a string quartet's parody of a choral work (SEE Figure 12). Next, examine the K. 546 setting for string quartet from this same vantage-point. Apply this same approach to the K. 475 Fantasy, up to as far (at least) as the allegro section (SEE Figure 13).

These suggested mental exercises, and analogous ones,

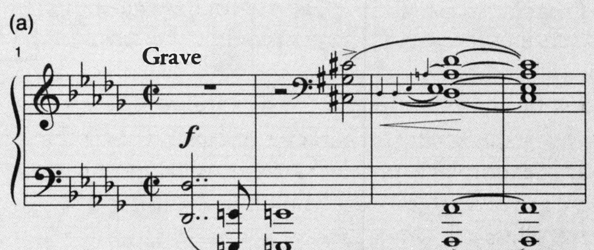


FIGURE 11. *The opening of the first movement of Frédéric Chopin's Sonata for Piano in B-flat minor, Op. 35, shown in (a), is a direct quotation from the opening of Beethoven's Sonata Op. 111, shown in (b).*

must tend to improve that quality of musical insight which borders upon recognition of the relevant thought-objects.[63] To this purpose, it will prove helpful to include in such a pedagogical program, emphasis upon post-1781 fugues and fugato composition of Mozart, Beethoven, and Brahms. Bach's work, presented in respect to his pivotal *Musical Offering* and *Art of the Fugue,* should be viewed in the post-1781 context; the post-1781 work by Haydn should then be included.

Once more, bring to bear the crucial point, that the generation of a musical thought-object occurs in essentially the same specific type of way that the appropriate solution is produced for the central paradox of Plato's *Parmenides:* all merely formal, discrete aspects of existence are subsumed by a higher mode of existence, *change.* The relevant, elementary form of this quality of change, is what we have described as "evolutionary negentropy." That point must be applied to define the crucial significance of the Bach fugue for the post-1781 work of Mozart *et al.*

Like a theorem-lattice series, the well-tempered counterpoint of Johann Sebastian Bach, has three prominent features. There is, first, the establishment of great refinement in constructing a formal musical theorem-lattice, the schoolbook side of studies of Bach's fugues, for example. Second, there is the creative development, like that of a science-discovery theorem-lattice, which generates the theories of paradox-resolutions which is the composition as a whole. Third, there is the effort to achieve a higher organic unity of the theorem-lattice series—the unit composition—as a whole, to subsume the *Many* as *One,* as Haydn sought this through his *Motivführung* discovery.

Thus, without all of the leading features of the work of the mature Johann Sebastian Bach, there could not have been Mozart's revolutionary perfecting of Haydn's *Motivführung* discovery. Even as extraordinary a genius as Mozart had become by 1781, could not have produced

FIGURE 12. *In December 1783, Mozart composed the Fugue in C (minor) for Two Pianos, K. 426, whose opening is shown in (a). The fact that his reference for the fugue's registration is a string quartet or vocal chorus, is unmistakable from his uncommon use of the vocal tenor clef for the left hand of Piano I, instead of the usual bass clef. The left hand of Piano II opens with the bass voice, while the right hands of Piano I and Piano II enter as mezzosoprano and soprano, respectively. Five years later, in the summer of 1788, Mozart re-scored the same fugue for string quartet, adding an*

the six "Haydn" quartets without a regular, extensive working-through of Bach scores which Mozart did, as a participant in the regular Sunday midday salon of Vienna's Baron Gottfried von Swieten [SEE article this issue, page 30].[64]

There are chiefly two relevant aspects of Bach's perfection of a *bel canto*-premised, strictly well-tempered polyphony (pivoted upon C = 256 cycles).[65] There is the formal side of Bach's contrapuntal method, the schoolbook side. There is, otherwise, that higher, creative treatment of lawfully generated contrapuntal anomalies, such as dissonances, a development whose mastery presumes a grounding in the formal, schoolbook side of the matter. On these combined accounts, the Mozart of 1782-1786 stands to the Bach of 1747-1750 as Nicolaus of Cusa of 1440 stood with respect to those manuscripts of ancient Archimedes freshly brought from Greece.

It is strict adherence to properly adduced formalities, which is a precondition for driving any theorem-lattice to beyond its limits, to such an effect that the appropriate, valid paradoxes are generated, and, so, the relevant creative discovery provoked. Thus, the notion of *Motivführung,* like the elementary form of a progressive series of theorem-lattices, presents us with a threefold picture of the creative process of unified compositional development:

1. Strict rigor respecting the formalities of polyphony, formalities broadly analogous to the consistency of the theorem-lattice.
2. The principle of those singularities which generate a new, higher formalism (e.g., theorem-lattice) out of a paradox generated within the original form. (These two paradoxes are parallel to those of the Plato *Parmenides.*)
3. The *Motivführung* principle, which orders, or implicitly subsumes an ordering of a succession of theorem-

(b)

adagio *introduction and calling it Adagio and Fugue in C (minor), K. 546. The corresponding opening measures of the re-scored fugue, shown in (b), show the standard string imitations of the vocal quartet: Violin I (soprano), Violin II (mezzosoprano), Viola (tenor), and Violoncello (bass). Also, Mozart has carefully altered the phrase markings and* staccato *(lightly accented and separated) markings to suit the particular requirements of the stringed instruments in order better to imitate the choral voices.*

FIGURE 13. *Mozart's Fantasy in C, K. 475, especially the sections up to the "allegro" (measures 36ff.), shows the composer's rigorous attention to "choral" registration of this piano work. Measures 15-18 shown here are representative. Compare with Figure 10, which shows Beethoven's direct quotation of these measures.*

lattices as an "evolutionary negentropy" unit of development.

Mozart's work on Bach, especially Bach's discovery represented by the *Musical Offering,* was necessary to generalize the third of these three features of an integrated compositional process. Only a rigorously defined, and ordered, *literate* medium of communication—geometry, music, poetry, or prose—provides the setting wanted to elaborate an anomaly in the needed fashion: to impart that sense of paradox which is associated with the creative-mental processes' successful generation of the relevant thought-object.

There is a second, crucial prerequisite to musical literacy. The lack of any first-rank, living Classical poets, since the generations of Goethe, Schiller, Keats, and Heine, is the cause of the loss—for most educated members of European civilization—or, at least a severe impairment, of the capacity to understand Classical polyphony. Not only is Classical polyphony derived from the *bel canto* vocalization of Classical poetry; the interrelationships, the continuing interdependence between the two forms, is such that to lose either one is virtually soon to lose the other.

As Friedrich Schiller, Ludwig van Beethoven, and Franz Schubert have emphasized this connection, in their common complaint against Goethe's refusal to tolerate the principles of Classical polyphony,[66] there is this stated essential reciprocity between the two. As Schiller stresses,[67] the composing of a Classical poem begins with an idea of wordless Classical polyphony in the imagination; the subsequent elaboration of this musical image, as poetic vocalization, defines the potential for the germination of the poem. So far as that, Goethe recognized the creation of Classical poetry to occur in this manner Schiller so indicated; Goethe's fault was his refusal to grasp the Platonic idea, that something like a *Motivführung* is indispensable to a fully developed Classical musical setting of a poem. Whoever could not follow that argument, with Goethe heading the one faction, and Schiller, Mozart, Beethoven, and Schubert the opposite faction, becomes, as a musician, like that amateur linguist who knows the meanings of none of those foreign-language phrases which he feigns to utter with such fluency.

For reason of such considerations, not only the singer, but the instrumentalist, too, must master this connection between Classical poetry and music, a study usefully pursued through the Italian art-song from Alessandro Scarlatti onward, and continued through that new form of German *Lied* established by Mozart's revolutionary "Das Veilchen."[68]

As this immediately foregoing argument is illustrated in Volume I of the *Manual on the Rudiments of Tuning and Registration,*[69] the practice of Mozart, Beethoven, Schubert, Schumann, and Brahms (most notably) in composing a *Lied* for a strophic poem[70] is the application of the Haydn-Mozart *Motivführung* principle of composition, as Mozart, chiefly, refined this. This is what Goethe and Reichardt[71] failed to comprehend. This feature of the *Lied,* from Mozart's "Das Veilchen," through Brahms' *Four Serious Songs,* is also a presentation of the essential characteristics of the *Motivführung* principle, the proper principle of all forms of successful Classical composition, and thus, also, the standard for performance of all such works from that same interval of musical history.

This view of Bach and Classical poetry has an associated benefit not to leave unmentioned here.

The principles of well-tempered polyphony are derived uniquely from those natural characteristics of the human singing voice which are made transparent by *bel canto* training. The setting of the well-tempered scale to values of approximately C = 256 and A = 430, is not a matter of whim; these values are derived from the biologically-determined spectroscopy of the "chest" of species of human singing voices. The musical system of well-tempered polyphony is not something externally applied to a poem, to generate a song; Classical poetry is composed, originally, in each case, under the governance of a literally musical idea in the mind of the poet. The vocalization of the poetic line inheres in the idea by which the line itself was originally generated.

Similarly, the definition of a dissonance, and its resolution, are so situated within, and premised upon a natural determination by a well-tempered polyphony. Well-tempered polyphony, at C = 256, or A = 430, is simply natural beauty, naturally determined. From this, artistic beauty begins, and to this it must return. In this way, Bach's perfection of well-tempered polyphony as a medium of composition provides the rigorous setting for such musical discoveries of higher principles as his own *Musical Offering,* and that for Mozart's revolutionary enhancement of Haydn's *Motivführung* principle.

Yet, that is not sufficient; the principles of well-tempered polyphonic development will not generate great music by themselves. All great composers returned to poetic text, or germs of poetic ideas, not only for their vocal, but also their instrumental works. All Classical musical thematic ideas are derived either from poetry, from original poetic ideas of the musical composer, or from the same type of a wordless idea of vocalization which is the germ of any Classical poem.

Except as we read the work of Mozart, Beethoven, *et al.* in the context both of Bach's development of well-tempered polyphony, and of all true music as an outgrowth of Classical poetry, there could be no genuine musical literacy among professionals or audiences. True musical literacy may be termed "insight," a term which addresses the shadows cast by the essential feature of Classical compositions, "Platonic ideas," otherwise termed "musical thought-objects."

Art Versus 'Materialism'

By means of description and references supplied, we have indicated, above, the nature of the common feature of scientific and artistic creativity. The immediate product of successful activity of this type, is the "thought-object," or *monad* treated here. As we have shown in earlier locations, this individual's creative mental activity is uniquely a *sovereign* experience of, and within the bounds of the individual mind; it is in no way a "collective" social effect.[72] In the case of such a valid discovery of a principle of physical science, the created thought-object *subsumes* a definite form of human practice. Immediately, this practice is expressed as an appropriate design of crucial experiment. This experimental (e.g., laboratory) design corresponds to *and subsumes* a consequent principle of machine-tool design. Such machine tools increase mankind's power over nature, *per capita* and per square kilometer. Thus, a "spiritual" act, the creation of such a thought-object, is an efficient causality in the (putatively) "material" domain.[73]

In the composition of Classical polyphony, the result is the same in principle. A problem—a paradox—generated by extended application of ostensibly consistent principles of well-tempered polyphony, provokes a musical thought-object. This process parallels Cusa's discovery of an isoperimetric least action. The generation of the solution, as a thought-object, is played back upon the polyphonic medium. The resolution so effected, is immediately analogous to a design of a crucial experiment. The elaboration of the newly discovered principle of resolution revolutionizes the power of polyphonic composition for entire works.

The point being made here is illustrated most aptly by introducing a contrasting reference to Descartes' *gnostic* dogma, *deus ex machina.*[74]

From the standpoint of mere sense-perception, a paradox in the sensory domain of experimental physics leads to a change in practice, an improvement, in the domain of experimental physics. Similarly, a musical paradox in the domain of tonal sense-perceptions leads to a resolution in the domain of tonal sense-perception. So, Descartes' (largely erroneous) mathematical physics, starts in the material domain and remains there, never departing; so, most formalist musicology situates musical theory. In both cases, the mechanistic, or "materialist" view either denies the existence of a creative process, or insists that cause-and-effect—problem, solution, and result—must all be fully explainable within the domain of sense-perception, never mentioning the creative-mental processes of problem-solving discovery, whether the latter might exist, or not. So, the majority of the most promising candidates for professional careers in physical science are crippled by the *gnostic* dogma, that science—problem, solution, result—must be explained (or, presumed to be explained) solely by means of "generally accepted classroom mathematics." The same pathological way of thinking, made officially canonical in musicology, has ruined the potential of musicians and audiences alike.

The material, or polyphonic domains, respectively, are each a realm of perception, of sense-perception, and of perceptible features of forms of social practice. Therefore, they are also the domains explicitly referenced by all forms of communication, including algebra and geometry. However, *causality does not occur within the domain of mere perception;* perception is not reality; it is merely the distorted shadow of reality. By "causality," we should not signify "mechanical" or "statistical" correlations; we should signify the cause of those types of change in state which are illustrated by the perceptibly efficient transformation of one theorem-lattice into another, perfectly inconsistent theorem-lattice.

Causality is thus presented to perception paradoxically, as this is presented in Plato's *Parmenides:* as change of this transfinite "dimensionality"; in this way, the efficiency, the reality, the ontological actuality of *change as causality* is presented with crucial undeniability to the faculties of sense-perception (and communication).

This causality, this change, is known to us in association with such various rubrics as "ideas" (Plato), "monads" (Leibniz), "*Geistesmassen*" (Riemann), or this author's "thought-objects." All of these terms reference the same phenomenon, but with slightly different connotations. The difference among them, is that each term was introduced by a different author, each in a unique literary-historical setting. Although all of these terms coincide in significance in the final analysis, their equivalence can be demonstrated only to those individual minds which have experienced all of them, one at a time, each in its own original setting.

For the subject of musical principles, three of these authors suffice. This present author's view of musical thought-objects is cross-referenced principally to the precedent of Platonic aesthetics, and, hence, Platonic ideas. In connection to the Haydn-Mozart revolution of 1781-1786, Friedrich Schiller's definitions of "musical thought-objects" should be included directly.[75]

In scientific and related work, the most profound distinction experienced by the individual, is the distinction between two qualities of mental state. The first state is the application of known, established principles; the second, is the act of discovery of a valid new principle, an act which occurs in the context of solving a true paradox. In music, it is the same; here, the act of discovering an insight into the characteristic idea of the composition's contrapuntal (polyphonic) development, is the creative state of mind.

It is the second of the two kinds of states of mental activity, which corresponds to the experiencing of a relevant thought-object, or thought-objects, as a species of mental life in general. Furthermore, in science and in Classical polyphony, these thought-objects are the cause for which a successful, problem-solving breakthrough to a valid new principle is the manifest consequence.

How is it possible, then, that so many from among even the highest echelons of achievement in modern science and the music profession should object so violently against "Platonic ideas," or be so stubbornly silly as to insist that these "spiritual" existences are not the cause for the new qualities of desired sense-perceptible effects? Since nothing less important than the continuation of human existence could not be achieved but by aid of such continuing scientific and technological progress, how could any self-respecting scientist deny the fact, that such "Platonic ideas" are the cause for manifest scientific progress?

Nonetheless, "Platonic ideas" are ruled out of order, not only by the "Aristotelian gnostic" René Descartes, but by the "materialists" and "empiricists" generally. These foolish denials are not a reflection of innocent sorts of ignorance; they are the influence of that form of modern pagan religion, of modern gnosticism, called the English and French "Enlightenment" of Europe's seventeenth and eighteenth centuries. The anti-Renaissance dogmas of Enlightenment figures such as Rosicrucian Robert Fludd, a co-founder of British Freemasonry,[76] and Descartes, became relatively hegemonic in today's classroom and popular opinion through such enterprises (often, London-backed) as France's Jacobin Freemasonic terror,[77] the 1815 Treaty of Vienna,[78] Lord Palmerston's Mazzinian terrorism of 1848-1849,[79] and Britain's authorship of World War I.[80] All of these, and related developments, were vehicles for efforts to crush out of existence Leibnizian science and to push aside the Classical tradition of Leonardo da Vinci, Raphael, Bach, Mozart, Schiller, and Beethoven in the fine arts.

To understand this aspect of the Enlightenment, two points must be stressed. First, the roots of the Rosicrucian cult in pre-Christian gnostic paganism, and such forerunners of Fludd, Francis Bacon, Descartes, Ashmole, John Locke, *et al.,* as the followers of Mani (Manicheanism) and the Bogomils-Cathars ("Buggers").[81] Second, that the common feature of ancient, medieval, and Rosicrucian gnostics, like Descartes and Immanuel Kant, too, is the emphasis upon denying the efficient, intelligible existence of "Platonic ideas."

Christian civilization defines a secular order in which all persons—all individual human life, is equal under God and natural law, this by virtue of the principle of individual man in *the living image of God (imago viva Dei).*[82] This likeness to the Creator is located in that "divine spark of reason," *creative mental powers,* inhering in each person; thus, is the person in the image of the Creator.[83] Thus, the domain of "Platonic ideas," *monads,* or "thought-objects," is the spiritual realm, while mere sensation and formal media of communication are the putative "material" realm.

The characteristic epistemological feature of all *gnosticism,* is the insistence that the spiritual realm has no desirable form of efficient (causal) interaction with the domain of the ostensibly "material." The gnostic dichotomy divides the universe into two universes, one "spiritual," the other "material," such that the events within each are defined entirely by laws (axioms, postulates) which are "hermetically," inclusively peculiar to the interior of that "half-universe."

So, the anti-Leibniz, neo-Aristotelian, Immanuel Kant, throughout his famous *Critiques,* pronounced *monads* "unknowable," and insisted that there is no principle of truth in the fine arts.[84] Kant's dogma was adopted by the nineteenth-century Romantic adversaries of Schiller and Classical polyphony, as the doctrine of the hermetic separation of *Geisteswissenschaft* (e.g., fine arts) from *Naturwissenschaft* (natural science).[85]

In the history of medieval and modern Europe, every significant spread of gnosticism is always associated with the promotion of Aristotle against Plato.[86] This is associated with a denial of a *Type* of activity[87] distinct as *creative,* and the axiomatic presumption that the internal ordering of the "material" realm is *algebraic* (i.e., mechanical). This Aristotelian, mechanistic view, applied to music, follows the pseudo-scientific tactic of Helmholtz's *Sensations of Tone,* purporting to explain music from the standpoint of a simply mechanistic dogma of percussion and vibrating strings and air.[88]

From medieval times, through the days of Paolo Sarpi,[89] Venice's Padua and Rialto schools, (together with the Isle of Capri of the former pagan Emperor Tiberius), were the center of radiation of the intertwined influences of Aristotle, gnosticism, and usury throughout Western Europe and into the Americas. Out of this influence, there emerged that "Venetian Party" which created British liberalism, and sought to make its captive colony, eighteenth-century Britain, the maritime base for building up a revived pagan Roman world-empire.[90] This "Venetian Party," with its sundry influences upon the continent of Europe, was the employer and sponsor of the gnostic Aristotelianism of Descartes, the seventeenth-century English Rosicrucians, and so on.

Thus, to this day, what we call "European culture," is not an homogenous culture, but rather a yet undecided, continuing war between Christianity, on the one side, and the powerful party of usury, the latter the pagan imperial faction behind the fostering of such gnostic Aristotelianisms as Rosicrucianism, Descartes, empiricism, Immanuel Kant, the nineteenth-century Romantic adversaries of Beethoven and Brahms, and so on.

The power of this gnostic, "Venetian Party" faction, has thus been the means for promoting the hegemony of materialism against both Leibnizian science and Classical fine art. Thus, for reason of that political hegemony of the gnostics in scientific and fine arts institutions, the appreciation of Classical fine art has been crippled. So, in fine arts, as in science, the Manichean dualism of Savigny's Romanticist separation of *Geisteswissenschaft* from *Naturwissenschaft* reigns.[91] So, the musicians learn the language of music, but are denied access to the meaning, the subject-matter of that fine-arts language.

The central issue is thus, that it is the product of creative reason, the musical thought-object, which employs the paradoxical implications of the sensory aspect of the polyphonic language, to impart a recognition of that same musical thought-object in the minds of others. The precious essence of Classical polyphony is in great danger of being lost to the next generations of mankind. The mission adopted by the crafters of the two-volume *Manual on the Rudiments of Tuning and Registration,* is to contribute to keeping that imperiled Classical fine-arts knowledge alive for both present and future generations.

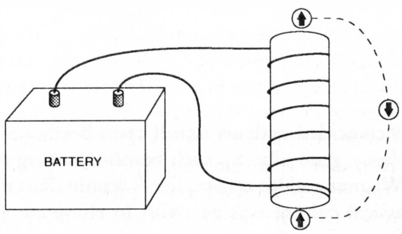
NOTES

1. Lyndon H. LaRouche, Jr., "The Classical Idea: Natural and Artistic Beauty," *Fidelio,* Vol. I, No. 2, Spring 1992.
2. Lyndon H. LaRouche, Jr., "On the Subject of Metaphor," *Fidelio,* Vol. I, No. 3, Fall 1992.
3. *A Manual on the Rudiments of Tuning and Registration* (Washington, D.C.: Schiller Institute, 1992), Vol. I, pp. 229-260.
4. LaRouche, "Metaphor," *op. cit.*
5. Joseph Haydn, *String Quartets Opus 20 and 33, Complete Edition,* ed. by Wilhelm Altmann (New York: Dover Publications, 1985). For a discussion of the influence of Haydn's *Motivführung* principle on Mozart's compositional method, see Hermann Abert, *W.A. Mozart, neubearbeitete und erweiterte Ausgabe von Otto Jahns Mozart* (Leipzig: VEB Breitkopf und Härtel, 1983), Vol. II, pp. 135-151.
6. J.S. Bach, *Musikalisches Opfer–Musical Offering–Offrance musicale,* ed. by Carl Czerny (New York: Edition Peters, No. 219).
7. Lyndon H. LaRouche, Jr., "Solution to Plato's Paradox: The 'One' and the 'Many,' " *Fidelio,* Vol. I, No. 1, Winter 1992, *passim.*
8. See footnote 3.
9. The Lord Palmerston-linked "Young Europe" insurrection of 1848-1849 coincided with an assault upon Beethoven and Classical polyphony generally, by such bomb-throwing anarchists as Richard Wagner and his accomplice Bakunin. Part of this assault upon Classical culture was an effort to eliminate an orchestral tuning of C = 256 cycles, by aid of redesigning wind instruments to fit the elevated pitch of A = 440 or higher.
10. To cite just three examples: (a) In 1787, Mozart reworked his Serenade No. 12 in C-minor for 2 Horns, 2 Oboes, 2 Clarinets, and 2 Bassoons, K. 388 (1782), as his Quintet in C-minor, for 2 Violins, 2 Violas, and Violoncello, K. 406. (b) In 1797, Beethoven reworked his Partita in E-flat for Wind Octet, Op. 103 (1792, published posthumously), as his Quintet for 2 Violins, 2 Violas, and Violoncello, Op. 4. (c) In 1801, the firm Mollo published Beethoven's Quintet in E-flat for Pianoforte and Wind Instruments, Op. 16, which he had composed in 1797, and simultaneously published Beethoven's own arrangement of the work as a Quartet for Pianoforte and Strings (not to be confused with a subsequently published arrangement for string quartet alone, which the composer had nothing to do with).
11. Cardinal Nicolaus of Cusa, *De Docta Ignorantia* (*On Learned*

Ignorance), trans. by Jasper Hopkins as *Nicholas of Cusa on Learned Ignorance* (Minneapolis: Arthur M. Banning Press, 1985), pp. 53-77; see also, "De Seculii Quadratura" ("On the Quadrature of the Circle"), trans. into German by Jay Hoffman (Mainz: Felix Meiner Verlag), *passim.*

12. Archimedes, "Measurement of a Circle," and "Quadrature of the Parabola," in *The Works of Archimedes,* ed by T.L. Heath (New York: Dover Publications), pps. 91-98, 233-252.
13. LaRouche, "Classical Idea," *op. cit.*
14. Plato discusses his theory of "ideas" (*eidē*) throughout the corpus of his dialogues, and the dialogue *Parmenides* is wholly devoted to its investigation. Primary locations, in assumed general chronology of composition, include: *Meno,* in *Plato: Laches, Protagoras, Meno, Euthydemus,* trans. by W.R.M. Lamb, 81b-87c; *Phaedo,* in *Plato: Euthyphro, Apology, Crito, Phaedo, Phaedrus,* trans. by H.N. Fowler, 72e-80d; *The Republic,* in *Plato: The Republic,* trans. by Paul Shorey, Vol. II, 505a-520a; *Parmenides,* in *Plato: Cratylus, Parmenides, Greater Hippias, Lesser Hippias,* trans. by H.N. Fowler, *passim.*; *Theatetus,* 184b-186e, and *The Sophist,* 248a-258c, both in *Plato: Theatetus and The Sophist,* trans. by H.N. Fowler. All editions are Loeb Classical Library (Cambridge: Harvard University Press); page numbers listed are used universally, however, and will appear as marginal notations in most editions.
15. Gottfried Wilhelm Leibniz, *Monadology,* trans. by George Montgomery (LaSalle: Open Court Publishing Co., 1989).
16. See Bernhard Riemann, "Zur Psychologie und Metaphysik," on Herbart's Göttingen lectures, for Riemann's reference to *Geistesmassen,* in *Mathematische Werke,* 2nd. ed. (1892), posthumous papers, ed. by H. Weber in collaboration with R. Dedekind.
17. LaRouche, "Metaphor," *op. cit.*
18. *Ibid.*
19. The topological aspect of the electromagnetic phenomenon is already evident in the simple solenoid experiment of Ampère's early researches: A.M. Ampère, *Theorie mathematique des phenomenes electro-dynamiques uniquement déduite de l'experience* (Paris: Blanchard, 1958).

    In the simple apparatus illustrated, the magnetic compass needle will be seen to rotate 360° in a 180° turn of the compass around the electrified solenoid, suggesting a multiply connected topology of action.



    Bernhard Riemann's investigations of toroidal and higher-genus topologies in connection with electrical "streamings" is reported in Felix Klein, *On Riemann's Theory of Algebraic Functions and Their Integrals,* trans. by Frances Hardcastle (Cambridge: MacMillan and Bowes, 1893).

    James Clerk Maxwell insisted that such topological features could be ignored for purposes of analysis, and that the higher-genus ("periphractic") regions of space could be reduced to simple connectedness by cuts ("diaphragms"): J.C. Maxwell, *A Treatise on Electricity and Magnetism* (New York: Dover, 1954), §18-22, 481.

    A devastating refutation of the entire theory of elasticity upon which the Maxwell electromagnetic theory was based, was given by Eugenio Beltrami in "Sull' equazioni generali dell' elasticitá" ("On the General Equations of Elasticity"), *Annali di Matematica pura ed applicata,* serie II, tomo X (1880-82), pp. 188-211; trans. by Richard Sanders, *21st Century Science & Technology,* unpublished.

20. The mathematician Hermann Grassmann constructed the putative mathematical proof for the Rupert Clausius/Lord Kelvin concoction known as the "Second Law of Thermodynamics," and was also employed by Clausius to concoct an incompetent criticism of Bernhard Riemann's work on electrodynamics.

    In an 1858 paper, *A Contribution to Electrodynamics,* Riemann asserted the coherence of the theory of electricity and magnetism with that of light and radiant heat, proposing that the electrodynamic effects are not instantaneous, but are propagated with constant velocity equal to the velocity of light. The paper was published posthumously and then criticized by Clausius, who objected to the appearance of an integral expressing the value of the potential, which he interpreted as capable of taking on an infinitesimally small value.

    A related criticism was made by Helmholtz against the work of Riemann's collaborator, Wilhelm Weber, the recognized leader in fundamental electrodynamic research. Helmholtz made the irresponsible charge that Weber's Law of Electrical Force contradicted the Law of Conservation of Force, by allowing two attracting charged particles to theoretically achieve an infinite *vis viva* (energy).

    Weber answered the criticism in his *Sixth Memoir on Electrodynamic Measurements,* trans. in *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science,* Vol. XLIII–Fourth Series, January-June 1872, pps. 1-20, 119-145. He pointed out that the objection was valid only if the charged particles were allowed an infinite velocity. Thus, Weber deduced that there must be a finite limiting velocity for two electrical particles, such that its square may not exceed c^2. Although Maxwell later renounced Helmholtz's attack in an edition of the *Treatise on Electricity and Magnetism,* the criticism is still found to this day.

    An English translation of Riemann's essay, accompanied by a sympathetic summary of Clausius' criticism by the German editor Heinrich Weber, is available in two locations: *International Journal of Fusion Energy,* Vol. 3, No. 1, January 1985, pp. 91-93; and also in Carol White, *Energy Potential* (New York: Campaigner Publications, 1977), pp. 295-300.

21. Lyndon H. LaRouche, Jr., "In Defense of Common Sense," in *The Science of Christian Economy and Other Prison Writings* (Washington, D.C.: Schiller Institute, 1991), pp. 8-41.
22. LaRouche, "Metaphor," *op. cit.,* pp. 20-22.
23. Plato, *Timaeus,* trans. by R.G. Bury, Loeb Classical Library (Cambridge: Harvard University Press, 1975), 54d-55d, pp. 131-135.
24. For Plato on geometry as dialectic, see Plato, *The Republic, op. cit.,* Book 7, 509d-543b.
25. See Nora Hamerman, "The Council of Florence: The Religious Event That Shaped the Era of Discovery," *Fidelio,* Vol. I, No. 2, Spring 1992, pp. 23-26.
26. Gottfried Wilhelm Leibniz, "Specimen Dynamicum" (1695), in *Leibniz Selections,* ed. by Philip P. Wiener (New York: C.S. Sons, 1951); Johann Bernoulli, "Curvatura Radii," in Diaphonous Nonformabus *Acta Eruditorum,* May 1697, trans. in D.J. Struik, ed., *A Source Book in Mathematics, 1200-1800* (Princeton, N.J.: Princeton University Press, 1968), pp. 391-399.
27. Nicolaus of Cusa, *De Docta Ignorantia, op. cit.,* pp. 53-66.
28. Luca Pacioli, *De Divina Proportione* (1497) (Vienna: 1896), for which Leonardo da Vinci drew the geometrical diagrams. Reproductions of these drawings appear in *The Unknown Leonardo,* ed. by Ladislao Reti (New York: McGraw-Hill Book Company, 1974), pp. 70-71.
29. See, for example, Johannes Kepler, *Mysterium Cosmographicum* (*The Secret of the Universe*), trans. by A.M. Duncan (New York: Abaris Books, 1981), p. 93: "For in one respect Nicholas of

Cusa and others seem to me divine, that they attached so much importance to the relationship between a straight line and a curved line and dared to liken a curve to God, a straight line to his creatures. . . ."

30. Pierre de Fermat, *Oeuvres Fermat,* ed. 1891, epistl. xlii, xliii.
31. Blaise Pascal, *L'oeuvre de Pascal,* ed. by Jacques Chevalier (Paris: Gallimard, 1954).
32. Christiaan Huygens, *The Pendulum Clock, or Geometrical Demonstrations Concerning the Motion of Pendula as Applied to Clocks,* trans. by Richard J. Blackwell (Ames: Iowa State University Press, 1986), *passim;* also, *Treatise on Light* (1690), trans. by Sylvanus P. Thompson (New York: Dover Publications, 1962).
33. See footnote 26.
34. In his lifetime, Leonardo was as famous as a musician as he was as an artist and engineer. Although the book *De Voce* (On the Voice) which Leonardo is presumed to have written is lost, the available codices provide crucial examples of his thinking, practice, and great influence upon the subsequent development of composition and design of stringed instruments.

    The most comprehensive reference is Emanuel Winternitz, *Leonardo da Vinci as a Musician* (New Haven: Yale University Press, 1982). Not only was Leonardo closely associated with the leading instrument makers of his day, but he was a celebrated virtuoso performer on the *lira da braccio,* a stringed, bowed instrument which is universally regarded as one of the closest forerunners of the violin. In its fully developed form it had a flat body, rounded shoulders, and five melody strings which could be stopped against the fingerboard, as well as two open strings that ran freely through the air outside the fingerboard and would sound only their full length when touched by the bow or plucked by the player's fingers. It was held against the upper arm, had a softer sound than the modern violin, and was used for polyphonic accompaniment (usually improvised) to the singing of poetry.

    Leonardo's interest in the design of instruments that could imitate and amplify vocal choral polyphony, is further exemplified by his work on inventing a "viola organista," a keyboard instrument analogous to the organ. Instead of producing the tones by wind, the "viola organista" used an arrangement by which the keys would activate a continuous bow across the strings, thus imitating an ensemble of viols.

    One of the earliest recorded musical inventions of Leonardo is a "lira" (presumably, a *lira da braccio*) in the unusual shape of a horse's skull, which he presented to the ruler of Milan, Ludovico Sforza, in 1482. This attempt to create a more resonant stringed instrument by utilizing the cavities of the skull, albeit in this case an animal skull, is highly suggestive with regard to Leonardo's perception of the relationship between sound production in the voice and in stringed instruments—especially since Leonardo was the first to identify, in his drawings of the human skull from around 1490, the sinus cavities which play a key role in defining registers and amplifying the voice.

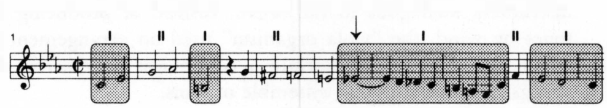
    The violin itself emerged at some point in the first half of the sixteenth century. In addition to omitting the two free strings, relative to the *lira da braccio* it reduced the number of melody strings to four and introduced the famous arched shape of the case, which gave the violin a capability of reproducing the intensity of the *bel canto* singing voice. In an essay reprinted in his 1967 book *Musical Instruments and their Symbolism in Western Art* (New Haven: Yale University Press, 1967), Winternitz presented the hypothesis that the first actual violin may be the invention of the painter Gaudenzio Ferrari, who depicts a clearly identifiable violin being played by an angel in a frescoed vault in Saronno, a town not far from Milan. While Gaudenzio was not a direct pupil of Leonardo, he was part of the Lombard school that had been shaped by Leonardo's influence during his two long sojourns in Milan, and he shared Leonardo's range of interests in painting, singing, and musical instrument development. Cremona, the city where the violin family of stringed instruments was perfected from the later sixteenth through the eighteenth centuries, is within this same Leonardo-influenced Lombard region. The imprint of Leonardo's influence on the visual arts is stamped throughout the region, and there is no reason to believe it would be any different for musical instruments.
35. In "The Vision of God" (1464), Nicolaus of Cusa develops the conception that each species, with its natural faculties as they develop, "yearns" for the existence of a higher species, as man does for the knowledge of the Absolute, of God. Here, Cusa's idea of negentropic species-evolution as the characteristic of Creation, is expressed by the poetic conception of *terminus specie.* The universe consists of negentropic growth of higher orderings, whose microcosm is human reason. The species recognizes this divine order of Creation, in its own way, and becomes a singularity in the transition from one ordering to the next. Thus, the species has a *terminus specie,* the actualization of infinity in one point, which enables further development.
36. LaRouche, "Metaphor," *op. cit.,* pp. 26-32.
37. See footnote 34.
38. Immanuel Kant, *Critique of Judgment,* trans. by J.H. Bernard (New York: Hafner Press, 1951), p. 152ff. For Friedrich Schiller's refutation of Kant, see Friedrich Schiller, "Letters on the Aesthetic Education of Man," in *Friedrich Schiller, Poet of Freedom,* ed. by William F. Wertz, Jr. (Washington, D.C.: Schiller Institute, 1985), Vol. I, pp. 251-255; and in "On Grace and Dignity," Vol. II (1988), pp. 365-368; "Aesthetical Lectures (1792-1793)," Vol . II (1988), pp. 471-481; "Kallias or, On the Beautiful," Vol. II (1988), pp. 482-526.
39. Formally, Plato's *eidos* is correctly translated as the English "idea"; in other words, Plato means what Leibniz identifies by *monads,* and I by "thought-objects."
40. Nicolaus of Cusa, *De Docta Ignorantia, op. cit.,* Book I.
41. Although the temple of the oracle of Delphi is usually identified with the cult of Apollo, even in Classical Greek times, Apollo was only one of the three pagan deities with which the complex was associated. The original deities of the site were, quite literally, Satan and his mother, known respectively by the local aliases, *Python* and *Gaia.* Python also used locally his Phrygian alias, *Dionysus.* In ancient times, through the time of the famous Delphi priest of Apollo, the biographer Plutarch, the oracle was a priestess who was assigned the name of *Pythia,* signifying her position as a priestess of Python. She delivered her utterances at the gravesite of Python-Dionysus. Later, after the service, the priests of Apollo provided the explanatory "spin" on the oracle's enigmatic messages. Python-Dionysus was equivalent to the Indian subcontinent's *Shiva,* the Semitic *Satan,* and the Hellenistic *Osiris;* this Dionysus was the Satan worshipped by that forerunner of New Ager Adolf Hitler, self-avowed anti-Christ, Friedrich Nietzsche. For Nietzsche's profession of being Dionysus the anti-Christ, see Friedrich Nietzsche, "Why I Am a Fatality" and *passim.* in "Ecce Homo," in *The Philosophy of Nietzsche* (New York: Modern Library, 1954), pp. 923-933.
42. The city of Rome rose to power among the Latins, and then in Italy, through the intervention of its patron the cult of Delphi. Roman legionnaires murdered Archimedes in 212 B.C.
43. See footnote 12. Cusa probably acquired his copy of Archimedes' writings from the Greek collection brought to Florence by George Gemistos ("Plethon").
44. For the work of Archimedes, see footnote 12. For a summary of the Egyptian method of squaring the circle, see Carl B. Boyer, *A History of Mathematics,* 2nd ed., revised by Uta C. Merzbach (New York: John Wiley & Sons, 1991), Chapter 2.

45. Nicolaus of Cusa, "De Circulii Quadratura," *op. cit.*
46. LaRouche, "Metaphor," *op. cit.,* pp. 18-20.
47. See below, Section IV.
48. See footnote 27.
49. Jacob Steiner, *Geometrical Constructions with a Ruler, Given a Fixed Circle with Its Center,* trans. by Marion Elizabeth Stark (New York: Scripta Mathematica, Yeshiva University, 1950). Steiner was Bernhard Riemann's instructor in geometry.
50. Euclid, *The Thirteen Books of Euclid's Elements,* trans. by T.L. Heath (New York: Dover Publications, 1956).
51. Lyndon H. LaRouche Jr., *A Concrete Approach to U.S. Science Policy* (Washington, D.C.: Schiller Institute, 1992).
52. See Carlo Zammattio, "The Mechanics of Water and Stone," in *The Unknown Leonardo, op. cit.,* pp. 190-207, for diagrams and citations to the various Leonardo manuscripts and codices; see also Dino De Paoli, "Leonardo: Father of Modern Science," in *Campaigner,* Vol. XV, No. 1, October 1985, pp. 34-37, for a review of Leonardo's investigations into fluid mechanics from a Riemannian standpoint. Leonardo's researches into hydrodynamics were assembled by F.L. Arconati in *Del moto e misura dell'acqua* (1643).
53. Johannes Kepler, *On the Six-Cornered Snowflake,* trans. and ed. by Colin Hardie (Oxford: Clarendon Press, 1966), reprinted by *21st Century Science & Technology,* 1991.
54. See footnote 25.
55. Georg Cantor, in *Georg Cantors Gesammelte Abhandlung,* ed. by Ernst Zermelow (Hildeschein, 1962); also, *Beiträge zur Begründungder transfiniten Mengenlehrer* (*Contributions to the Founding of the Theory of Transfinite Numbers*), trans. by Philip E.B. Jourdain (New York: Dover Publications, 1955), pp. 282-356.
56. In 1931, the Austrian mathematician Kurt Gödel demonstrated, by formal means, that one can formulate propositions within a formal logical system, the truth of which cannot be determined within the rules of that system. Gödel's proof served as an answer, in the negative, to the "Second Problem" of the famous twenty-three unsolved problems which Göttingen University mathematician David Hilbert had proposed in 1900 to the Second International Mathematical Congress in Paris. Hilbert's "Second Problem" was to determine whether it can be proved that the axioms of arithmetic are consistent—that is, can never lead to contradictory results. The same formal premise lay behind many of Hilbert's questions, including the "Tenth Problem," which concerns the solvability of Diophantine equations (algebraic equations in which the coefficients and solutions must be integers). For Hilbert's "Tenth Problem," see Carl B. Boyer, *A History of Mathematics, op. cit.,* pp. 610-614. See also Ernest Nagel and James R. Newman, *Gödel's Proof* (New York: New York University Press, 1958).
57. In 1672, Gottfried Leibniz was appointed to what we would term today a "fellowship" to minister Jean-Baptiste Colbert's Paris-based French Royal Academy of Science, where he began his long association with Christiaan Huygens.
58. For example, Huygens made use of Leonardo's construction of the aberration of light in a spherical mirror, in the closing pages of his *Treatise on Light* (New York: Dover Publications, 1962), p. 127.
59. Gottfried Wilhelm Leibniz, "History and Origin of the Differential Calculus," in *The Early Mathematical Manuscripts of Leibniz,* trans. by J.M. Child (LaSalle: Open Court Publishing Co., 1920).
60. The essential features of the author's 1948-1952 discoveries are restated within "On the Subject of Metaphor," *op. cit.*
61. It is a fact cut, quite literally, in stone, that the teaching of *bel canto* to church choirs was well established in Florence, Italy before the 1430's. The 1431 sculptures by Luca del Robbia in the choir stalls of the Florence cathedral Santa Maria del Fiore, shows the children singing in the mode we know today as the Florentine *bel canto.* Unfortunately, during the seventeenth and eighteenth centuries, a pseudo-*bel canto* raised in Venice and elsewhere, a "Venetian *bel canto*" design for *castrati* not recommended for would-be tenors today. See Nora Hamerman, *op. cit.,* and unpublished research on the Venetian pseudo-*bel canto.*
62. J.S. Bach's *A Musical Offering* consists of two major fugal investigations of the "royal theme"—so named because it was given to him by King Frederick "The Great" of Prussia—along with a number of canonical demonstrations, and a full trio sonata. In the first fugal investigation, the "Three-Part Ricercar" ("*ricercar*" = research or investigation), Bach presents the theme in the soprano voice:

The vocal register indications have been added according to the convention established in *A Manual on Tuning, op. cit.* The third register (III) is indicated by an unfilled box with a thick-shaded outline, the second register (II) is left unmarked, and the first register (I) is denoted either by a filled shaded box (in female voices), or by an unfilled thin-outline box (in male voices).

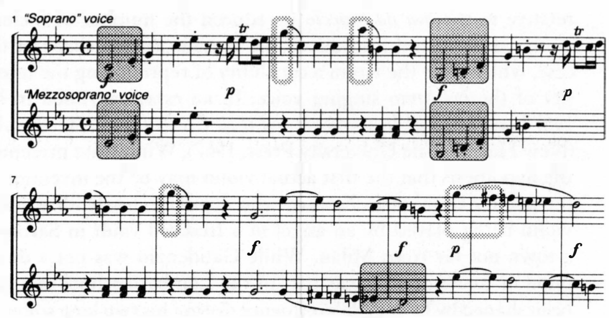
The theme opens with two notes in the second register, followed by two in the third, and then a steep drop back into the second register on the B♮. The fourth measure then focuses squarely on the III-II register shift by having F♯ on the first, most-emphasized beat, immediately followed by the F♮. The phrase continues downward in the second register, moving by the smallest possible step, the half-step, concluding with a jump to a final cadence.

In contrast to the "Three-Part Ricercar," in the "Six-Part Ricercar" Bach introduces the theme in the *mezzosoprano* voice:



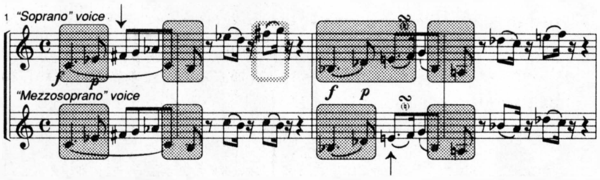
The registration of the theme's first five notes remains similar to that of the soprano; but the registration of the descending figure which follows, shifts attention to the theme's built-in ambiguity between the C-major mode, with its E♮ as the third degree of the scale, and C-minor, whose third degree is lowered by a half-step to E♭. This major-minor crossover ambiguity provides the rudimentary thought-object which drives the development of the entire *Musical Offering* series.

The opening measures of Mozart's Sonata for Piano in C-minor, K. 457, demonstrates Mozart's advance in the treatment of the same thematic idea:



Only the "soprano" and "mezzosoprano" voices in the piano score are shown here. The first five notes are sung in unison by both voices, once again with similar registration. Only in measures 9-13 is the crossover ambiguity presented. The descending mezzosoprano figure is answered by an octave transposition of the same descending figure in the soprano voice.

Mozart subsequently composed his Fantasy in C, K. 475, expressly in order to explicate the principles of his composition of the Sonata K. 457. The opening measures show the ambiguities of the "royal theme" in a most concentrated form:



Once again, only the "soprano" and "mezzosoprano" lines of the piano score are shown. The opening unison phrase now presents *both* the F♯ and the E♭, which taken together constitute a "limit" beyond which the registrations would cease to be similar. The second measure is dominated by soprano registration, with the high F♮. The third and fourth measures, however, are instead dominated by mezzosoprano registration, with its register shift (from below) to the E♮. The poetic shift from the first pair of measures to the second pair is underlined by the phrase markings in measure 3, which differ from those in the first measure. (Many modern editions of Mozart's piano works have mistakenly altered Mozart's phrase markings to be identical in meaures 1 and 3.)

63. In the author's judgment, the relevant musical thought-object is made clear by extended concentration on hearing the performance of the score heard, repeatedly, with experimental variation, in one's imagination.
64. On Baron Gottfried von Swieten and his salon, see David Shavin, "Mozart and the American Revolutionary Upsurge," article this issue; also see Bernhard Paumgartner, *Mozart* (München: 1991), pp. 299-308.
65. Jonathan Tennenbaum, "The Foundations of Scientific Musical Tuning," *Fidelio,* Vol. I, No. 1, Winter 1992.
66. See *A Manual on Tuning, op. cit.,* chap. 11, *passim.*
67. *Ibid.,* p. 201, footnotes 2-5.
68. *Ibid.,* pp. 202-208.
69. *Ibid.,* pp. 208-220.
70. See Gustav Jenner, *Johannes Brahms als Mensch, Lehrer und Künstler, Studien und Erlebnisse* (Marburg an der Lahn: N.G. Elwert'sche Verlagsbuchhandling, G. Braun, 1930). Selected passages appear in *A Manual on Tuning, op. cit.,* chaps. 9-12, *passim.*
71. See *A Manual on Tuning, op. cit.,* chap. 11, *passim.*
72. LaRouche, "Metaphor," *op. cit.,* p. 41; see also "The Science of Christian Economy," in *Christian Economy, op. cit.,* pp. 229-240.
73. LaRouche, "Metaphor," *op. cit.,* pp. 36-37.
74. LaRouche, "Metaphor," *op. cit.,* pp. 37-39; see also *U.S. Science Policy, op. cit.,* chap. IV, pp. 108-111 and footnote 3.
75. See *A Manual on Tuning, op. cit.,* chap. 11, *passim.*
76. LaRouche, "The Science of Christian Economy," in *Christian Economy, op. cit.,* p. 482.
77. Terror demagogues Danton and Marat were trained and deployed by London, under the immediate supervision of the Earl of Shelburne's (British East India Company's) Jeremy Bentham. The patronage of Robespierre's circles was provided jointly by the London-allied figures Philippe "Egalité," Duke of Orleans, a leading Freemason, and Swiss banker Jacques Necker, who had bankrupted the French monarchy's government. Necker's daugher, the notorious Madame de Staël, a putative friend of Queen Marie Antoinette, conducted the fashionable salon through which the political cause of the Jacobin butchers was greatly assisted.
78. The British government, acting through the Treaty of Vienna's Bourbon Restoration, purged France's leading scientific institution, the Ecole Polytechnique, of its founder Gaspard Monge, and of Monge's brilliantly successful Leibnizian program of education and work. French science collapsed rapidly then, to the point, that from approximately 1827 on, Germany became the world's leader in science—until Adolf Hitler's time.
79. Lord Palmerston, as Britain's Prime Minister, placed his protégé, Napoleon III, into power in France, as a continuation of Palmerston's earlier deployment of the Mazzinian Freemasonic terror of 1848-49 throughout continental Europe.
80. LaRouche, *U.S. Science Policy, op. cit.,* chap. IV, pp. 103-107.
81. *Ibid.,* chap. IV, pp. 93-97.
82. LaRouche, "The Science of Christian Economy," in *Christian Economy, op. cit.,* pps. 224-236, 301-303, 432-439.
83. *Cf.* Philo ("Judaeus") of Alexandria, "On the Account of the World's Creation Given by Moses," in *Philo,* Vol. I., trans. by F.H. Colson and G.H. Whitaker, Loeb Classical Libary (Cambridge: Harvard University Press, 1981), §XXIII, pp. 55-57.
84. Immanuel Kant, *Critique of Judgment, op. cit., passim.* See also LaRouche, "The Science of Christian Economy," in *Christian Economy, op. cit.,* pp. 333-334.
85. Berlin University law professor Karl S. Savigny, forerunner of the Nazi legal dogma, was a leading nineteenth-century spokesman for the Romantics' irrationalist dogma toward both art and science. He put into currency today's commonly taught, neo-Kantian dogma asserting an "hermetic" separation of *Geisteswissenschaft* from *Naturwissenschaft.*
86. This began in the Eastern hierarchy of the Church, under the direction of the Byzantine Emperors; there, the banning of Plato in favor of Aristotle was established many centuries before this gnostic dogma was inserted into Western Europe via Moorish Spain and Venice. Of course, the so-called Neoplatonic cults, which were developed in Byzantium and transported into Western Europe, were actually products of Aristotelianism, not Plato.
87. The term, "*Type,*" is used here in Georg Cantor's sense.
88. Hermann L.F. Helmholtz, *On the Sensations of Tone as a Physiological Basis for the Theory of Music,* 2nd. English edition, trans. by Alexander J. Ellis (New York: Dover Publications, 1954).
89. Paolo Sarpi (1550-1623) was a former Procurator-General of the Servite religious order, who in 1606 was appointed state theologian of Venice on the eve of a bitter fight between Venice and the Catholic Church. He was a leading theoretician of the "new houses" (*i nuovi*) of the Venetian aristocracy, which took ascendancy against the "old houses" (*i vecchi*) in 1582, in one of the most dramatic power struggles in Venetian history.

    The *nuovi* faction proposed: (1) an all-out assault against the Church at Rome, and Rome's allies, Spain and the Hapsburg dynasty; and (2) a major redeployment of Venetian financial power north into England and Holland, given the discovery of the New World and the opening of new trade routes. Although he was a radical materialist and apologist for the vast Venetian family fortunes then being reorganized, Sarpi as Venice's ideological hatchetman excoriated the Catholic Church as "worldly," "corrupt," and ruled by a "papal monarchy."
90. H. Graham Lowry, *How the Nation Was Won: America's Untold Story, 1630-1754* (Washington, D.C.: Executive Intelligence Review, 1987), pps. 74-76, 158-201.
91. See Andreas Buck, "Das Elend der deutschen Jurisprudenz: Karl von Savigny," *Ibykus,* Vol. III, No. 11, 1984, pp. 47-54.