The Foundations of Scientific Musical Tuning

by Jonathan Tennenbaum

I want to demonstrate why, from a scientific standpoint, no musical tuning is acceptable which is not based on a pitch value for middle C of 256Hz (cycles per second), corresponding to A no higher than 432Hz. In view of present scientific knowledge, all other tunings including A=440 must be rejected as invalid and arbitrary.

Those in favor of constantly raising the pitch typically argue, "What difference does it make what basic pitch we choose, as long as the other notes are properly tuned *relative* to that pitch? After all, musical tones are just frequencies, they are all essentially alike. So, why choose one pitch rather than another?" To these people, musical tones are like paper money, whose value can be inflated or deflated at the whim of whoever happens to be in power.

This liberal philosophy of "free-floating pitch" owes its present power and influence in large part to the acoustical theories of Hermann Helmholtz, the nineteenth-century physicist and physiologist whose 1863 book, *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (*The Theory of the Sensations of Tone as a Foundation of Music Theory*) became the standard reference work on the scientific bases of music, and remains so up to this very day. Unfortunately, every essential assertion in Helmholtz's book has been proven to be false.

This article is based on a speech given by the author, Director of the European Fusion Energy Foundation, at an April 1988 Schiller Institute conference on scientific tuning held in Milan, Italy. It appears also in the Institute's "Manual on the Rudiments of Tuning and Registration."

Helmholtz's basic fallacy—still taught in most music conservatories and universities today—was to claim that the scientific basis of music is to be found in the properties of vibrating, inert bodies, such as strings, tuning forks, pipes, and membranes. Helmholtz defined musical tones merely as periodic vibrations of the air. The fundamental musical tones, he claimed, are sine waves of various frequencies. Every other tone is merely a superposition of added-up sine waves, called "overtones" or "harmonics." The consonant musical intervals are determined by properties of the "overtone series" to be simple whole-number ratios of frequencies. Arguing from this standpoint, Helmholtz demanded that musicians give up well-tempering and return to a "natural tuning" of whole-number ratios; he even attacked the music of J.S. Bach and Beethoven for being "unnatural" on account of their frequent modulations.

Helmholtz based his theory of human hearing on the same fallacious assumptions. He claimed that the ear works as a passive resonator, analyzing each tone into its overtones by means of a system of tiny resonant bodies. Moreover, he insisted that the musical tonalities are all essentially identical, and that it makes no difference what fundamental pitch is chosen, except as an arbitrary convention or habit.

Helmholtz's Theory: Linear and Wrong

Helmholtz's entire theory amounts to what we today call in physics a "scalar," "linear," or at best, "quasi-linear" theory. Thus, Helmholtz assumed that all physical magnitudes, including musical tones, can at least implicitly be measured and represented in the same way as lengths along a straight line. But, we *know* that every important aspect of music, of the human voice, the human mind, and our universe as a whole, is characteristically *nonlinear.* Every physical or aesthetic theory based on the assumption of only linear or scalar magnitudes, is bound to be false.

A simple illustration should help clarify this point. Compare the measurement of lengths on a straight line with that of arcs on the circumference of a circle. A straight line has no intrinsic measure; before we can measure length, we must first choose some unit, some interval with which to compare any given segment. The choice of the unit of measurement, however, is purely arbitrary.

The circle, on the contrary, possesses by its very nature an intrinsic, *absolute* measure, namely one complete cycle of rotation. Each arc has an absolute value as an *angle,* and the regular self-divisions of the circle define certain specific angles and arcs in a lawful fashion (e.g., a right angle, or the 120° angle subtended by the side of an equilateral triangle inscribed in the circle).

Just as the process of rotation, which creates the circle, imposes an absolute metric upon the circle, so also the process of creation of our universe determines an absolute value for every existence in the universe, including musical tones. Helmholtz refused to recognize the fact that our universe possesses a special kind of curvature, such that all magnitudes have absolute, geometrically-determined values. This is why Helmholtz's theories are systematically wrong, not merely wrong by accident or through isolated errors. Straight-line measures are intrinsically fallacious in our universe.

For example, sound is not a vibration of the air. A sound wave, we know today, is an electromagnetic process involving the rapid assembly and disassembly of geometrical configurations of molecules. In modern physics, this kind of self-organizing process is known as a "soliton." Although much more detailed experimental work needs to be done, we know in principle that different frequencies of coherent solitons correspond to distinct geometries on the microscopic or quantum level of organization of the process. This was already indicated by the work of Helmholtz's contemporary, Bernhard Riemann, who refuted most of the acoustic doctrines of Helmholtz in his 1859 paper on acoustical shock waves.[1]

Helmholtz's theory of hearing also turned out to be fallacious. The tiny resonators he postulated do not exist. The human ear is intrinsically nonlinear in its function, generating singularities at specific angles on the spiral chamber, corresponding to the perceived tone. This is an active process, akin to laser amplification, not just passive resonance. In fact, we know that the ear itself *generates* tones.

Moreover, as every competent musician knows, the simple sinusoidal signals produced by electronic circuits (such as the Hammond electronic organ) *do not* constitute musical tones. Prior to Helmholtz, it was generally understood that the *human singing voice,* and more specifically, the properly trained *bel canto* voice, is the standard of all musical tone. Historically, all musical instruments were designed and developed to imitate the human voice as closely as possible in its nonlinear characteristics.

The *bel canto* human voice is for sound what a laser is for light: The voice is an *acoustical laser,* generating the maximum density of electromagnetic singularities per unit action. It is this property which gives the *bel canto* voice its special penetrating characteristic, but also determines it as uniquely beautiful and uniquely musical. By contrast, electronic instruments typically produce Helmholtzian sine-wave tones, which are ugly, "dead," and unmusical exactly to the extent that they are incoherent and inefficient as electromagnetic processes.

Tuning Is Based on the Voice

The human voice defines the basis for musical tuning and, indeed, for all music. This was clearly understood long before Helmholtz, by the scientific current associated with Plato and St. Augustine, and including Nicolaus of Cusa, Leonardo da Vinci and his teacher Luca Pacioli, and Johannes Kepler. In fact, Helmholtz's book was a direct attack on the method of Leonardo da Vinci.

If Helmholtz's theories are wrong, and those of Plato through Kepler and Riemann have been proven correct—at least as far as these went—then what conclusions follow for the determination of musical pitch today? Let us briefly outline the compelling reasons for

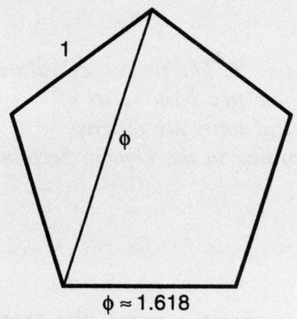


FIGURE 1. *The Golden Section arises as the ratio of the side to the diagonal of a pentagon.*

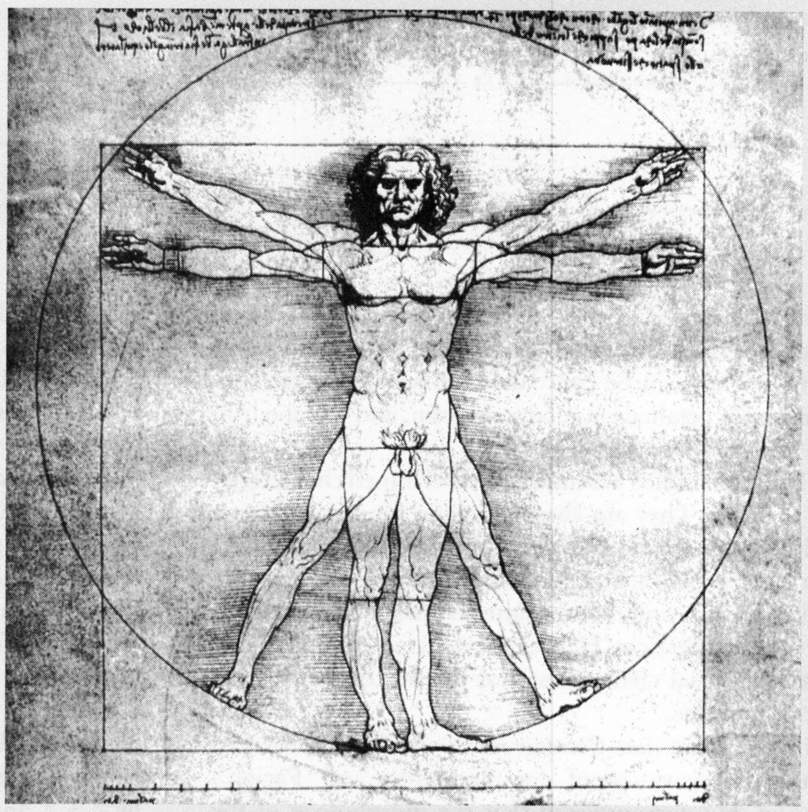


FIGURE 2. *Leonardo da Vinci's drawing of the human body inscribed in a circle demonstrates Golden Section proportions.*

C=256Hz as the only acceptable scientific tuning, which have emerged from a review of the classical work of Kepler *et al.* as well as modern scientific research.

The human voice, the basic instrument in music, is also a living process. Leonardo and Luca Pacioli demonstrated that all living processes are characterized by a very specific internal geometry, whose most direct visible manifestation is the morphological proportion of the Golden Section. In elementary geometry, the Golden Section arises as the ratio between the side and the diagonal of a regular pentagon (see Figure 1). The Golden Section naturally forms what we call a self-similar geometric series—a growth process in which each stage forms a Golden Section ratio with the preceding one. Already before Leonardo da Vinci, Leonardo Pisano (also called Fibonacci) demonstrated that the growth of populations of living organisms always follows a series derived from the Golden Section. In extensive morphological studies, Leonardo da Vinci showed that the Golden Section is the essential characteristic of construction of *all* living forms. For example, Figure 2 illustrates the simplest Golden Section proportions of the human body.

Since music is the product of the human voice and human mind—i.e., of living processes—therefore, everything in music must be coherent with the Golden Section. This was emphatically the case for the development of Western music from the Italian Renaissance up through Bach, Mozart, and Beethoven.

The Classical well-tempered system is itself based on the Golden Section. This is very clearly illustrated with the following two series of tones, whose musical significance should be evident to any musician: C–E♭–G–C, and C–E–F♯–G. In the first series, the differences of the frequencies between the successive tones form a self-similar series in the proportion of the Golden Section. The frequency differences of the second series decrease according to the Golden Section ratio (see Figure 3).

The Golden Section

To understand the well-tempered system better, we must first examine the reason why certain specific proportions, especially the Golden Section, predominate in our universe, whereas others do not.

There is nothing mysterious or mystical about the appearance of the Golden Section as an "absolute value" for living processes. Space itself—that is, the visual space in which we perceive things—has a specific "shape" coherent with the Golden Section. For, space does not exist as an abstract entity independent of the physical universe, but is itself *created*. The geometry of space reflects the characteristic curvature underlying the pro-

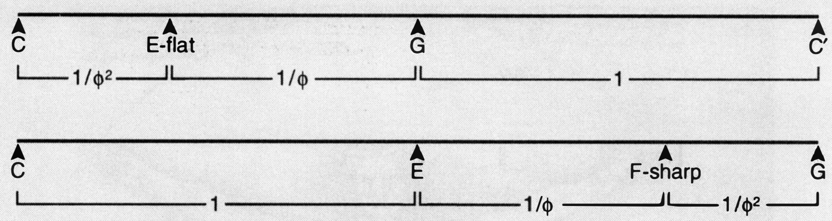


FIGURE 3. *The frequency values of these two basic series of musical tones are ordered according to the Golden Section.*

cess of generation of the universe as a whole. We know that space has a specific shape, because only five types of regular solids can be constructed in space: the tetrahedron, cube, octahedron, dodecahedron, and icosahedron (see Figure 4).

These five solids are uniquely determined characteristics of space. They are absolute values for all of physics, biology, and music. Indeed, Luca Pacioli emphasized that all the solids are derived from a single one, the dodecahedron, and that the latter is uniquely based upon the Golden Section. Hence, the Golden Section is the principal visual characteristic of the process of creation of the universe.

In his *Mysterium Cosmographicum,* Kepler provided further, decisive proof for Leonardo and Pacioli's method. He demonstrated that the morphology of the solar system, including the proportions of the planetary orbits, is derived from the five regular solids and the Golden Section. Figure 5 shows Kepler's famous construction of the planetary orbits through a nested series of concentric spheres, whose spacing is determined by inscribed regular solids. Therefore, the solar system has the same morphological characteristics as living organisms.

Kepler located the underlying reason for these morphological characteristics in the generating process of the universe itself, and this he attempted to identify with the help of what is called the *isoperimetric theorem.* This theorem states that among all closed curves having a given parameter, the circle is the unique curve which encloses the greatest area. Circular action is the maximally efficient form of action in visible space, and therefore coheres uniquely with the *bel canto* musical tone and the beam generated by a laser. Kepler reasoned that if circular action reflects uniquely the creative process of the universe, then the form of everything which exists—of atoms and molecules, of the solar system, and the musical system—must be constructible using nothing but circular action.

By this procedure, called "synthetic geometry," we generate from the circle, by folding it upon itself (i.e., circular action applied to itself), a straight line, the diameter. By folding again, we obtain a point, the center of the circle, as the intersection of two diameters, as in Figure 6. This alone creates for us the basic "elements" of plane geometry. Also, by rotating a circle we obtain the sphere (see Figure 7).

Further constructions, using circular action alone, generate the regular polygons—the equilateral triangle, square, and pentagon—which form the faces of the five regular solids. From these uniquely determined polygons, Kepler derived the fundamental musical intervals of the fifth, fourth, and major third, *without any reference to overtones.* These polygons embody the principle of self-division of circular action by 3, 4, and 5. The octave, or division by 2, we already obtained as the very first result of folding the circle against itself. From division by 2, 3, 4, and 5 we obtain, following Kepler, the following values for the basic musical intervals: octave, 1:2; fifth, 2:3; fourth, 3:4; major third, 4:5.

Division by seven is invalid, Kepler argued, because the heptagon is not constructible from circular action alone, nor does it occur in any regular solid. Since Kepler's musical ratios are uniquely coherent with the regular solids, they are uniquely coherent with the Golden Section underlying those solids.

Kepler went on to demonstrate that the angular velocities of the planets as they move in their elliptical orbits around the sun, are themselves proportioned according to the same ratios as the fundamental musical intervals (see Table I, page 56). Since Kepler's time, similar relations have been demonstrated in the system of moons of various planets, and provisionally also even in the motion of spiral galaxies.

C=256 As a 'Keplerian Interval'

C=256 has a uniquely defined astronomical value, as a Keplerian interval in the solar system. The period of one cycle of C=256 ($1/256$ of a second) can be constructed as follows. Take the period of one rotation of the Earth. Divide this period by 24 ($=2\times3\times4$), to get one hour. Divide this by 60 ($=3\times4\times5$) to get a minute, and again by 60 to obtain one second. Finally, divide that second

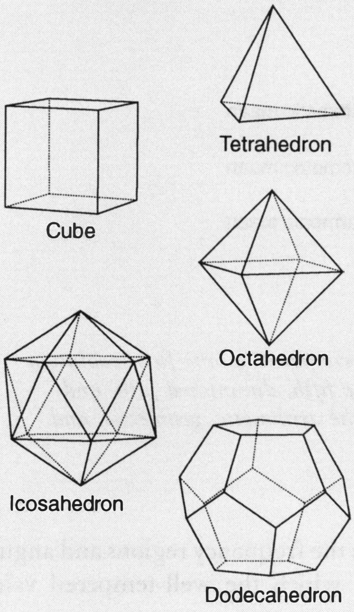


FIGURE 4. *The five Platonic solids.*

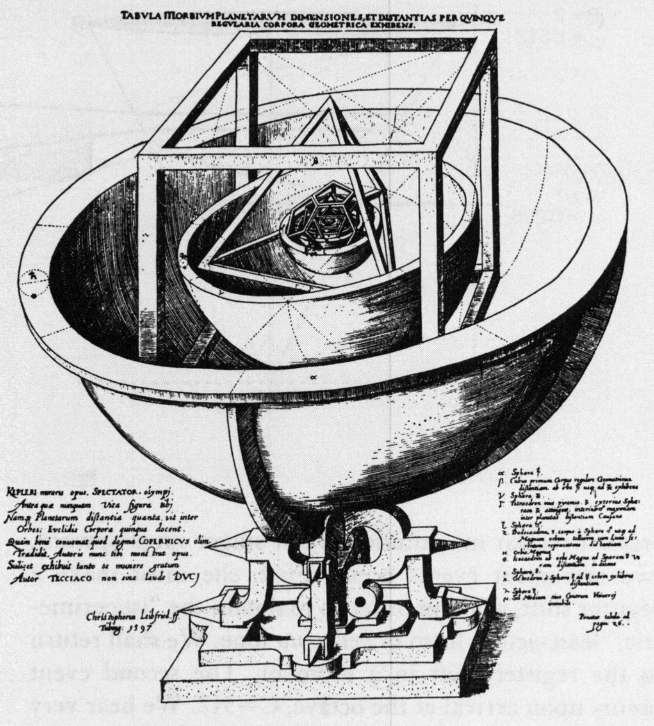


FIGURE 5. *Kepler's construction of the planetary orbits, nested according to a series of inscribed regular solids.*

by 256 (=2×2×2×2×2×2×2×2). These divisions are all Keplerian divisions derived by circular action alone. It is easy to verify, by following through the indicated series of divisions, that the rotation of the Earth is a "G," twenty-four octaves lower than C=256. Similarly, C=256 has a determinate value in terms of the complete system of planetary motions.

By contrast, A=440 is a purely arbitrary value, having no physical-geometrical justification. A=440 is an insane tuning in the rigorous sense that it bears no coherent relationship with the universe, with reality.

Today, we can add some essential points to this. Kepler's solution was absolutely rigorous, as far as it went; however, circular action is only an incomplete representation of creative action in the universe. The next great step was taken by Carl Friedrich Gauss at the beginning of the nineteenth century. Gauss introduced *conical spiral action,* instead of mere circular action, as the basis for synthetic geometry. Spiral action combines the isoperimetric principle of the circle with the principle of growth expressed by the Golden Section.

Let us demonstrate conical spiral action in the *bel canto* voice. Have a soprano sing a scale upward, starting at middle C (=256). As the frequency increases, so does the intensity of the sound produced. The more precise term for this intensity is "energy flux density." But this increase is not merely linear extension, not merely in-

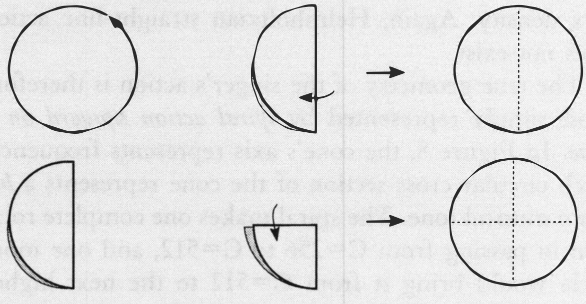


FIGURE 6. *Using "synthetic geometry," we generate a straight line and a point from the circle, by folding it upon itself.*

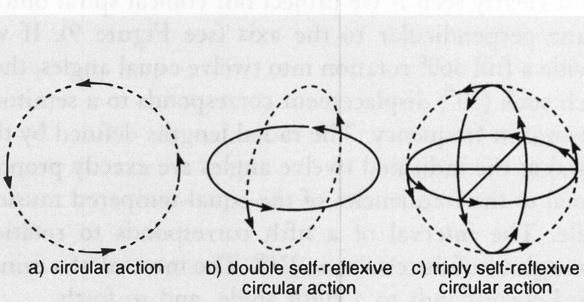


FIGURE 7. *The sphere is generated by triply-connected circular action.*

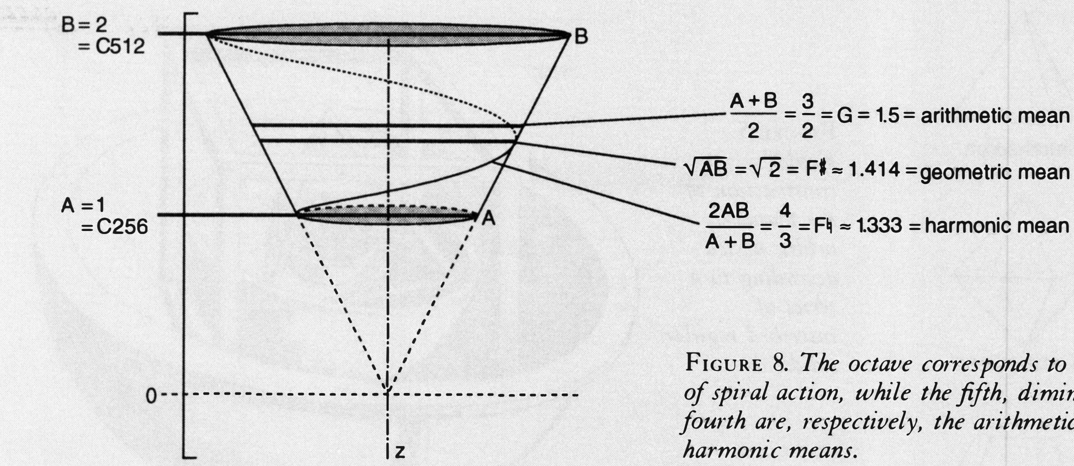


FIGURE 8. *The octave corresponds to one full revolution of spiral action, while the fifth, diminished fifth, and fourth are, respectively, the arithmetic, geometric, and harmonic means.*

crease in scalar magnitude. As our singer sings upward, two important events occur. First, she must make a register shift, at F♯, in order to maintain the "isoperimetric," least-action form of *bel canto* tone. We shall return to the register shift in a moment. The second event occurs upon arrival at the octave, C=512. We hear very clearly, that *one cycle of action* has been completed, like a 360° rotation. This proves that there is a *rotational* component of action to increase the frequency or energy flux density. Again, Helmholtzian straight-line action does not exist.

The true geometry of the singer's action is therefore most simply represented by *spiral action upward on a cone.* In Figure 8, the cone's axis represents frequency. Each circular cross-section of the cone represents a *bel canto* musical tone. The spiral makes one complete rotation in passing from C=256 to C=512, and one more cycle would bring it from C=512 to the next higher octave, C=1024. Thus, the interval of an octave corresponds to one complete 360° cycle of conical spiral action.

Not only the octave, but all musical intervals, correspond to specific angles on conical-spiral action. This is most clearly seen if we project our conical spiral onto a plane perpendicular to the axis (see Figure 9). If we divide a full 360° rotation into twelve equal angles, then each such (30°) displacement corresponds to a semitone interval in frequency. The radial lengths defined by the spiral at the indicated twelve angles are exactly proportional to the frequencies of the equal-tempered musical scale. The interval of a fifth corresponds to rotation through 7/12 of the circle, or 210°. The interval of a minor third corresponds to a right angle, and so forth.

(The equal-tempered system is only an approximation of a rigorous well-tempered system whose details have yet to be fully elaborated. Nevertheless, the indicated construction identifies the frequency regions and angular displacements within which the well-tempered values are to be defined.)

Most important, the halfway point of the full cycle starting at C, is F♯, the diminished fifth from C, or the interval once known as the "devil's interval." In terms of geometrical proportion, this F♯ is located as the *geometric mean* of C=256 and its octave, C=512.

If we carry out synthetic geometry constructions with conical spiral action, just as Kepler did with circular action, we discover wonderful things. For example, construct the characteristic of the conical volume bounded

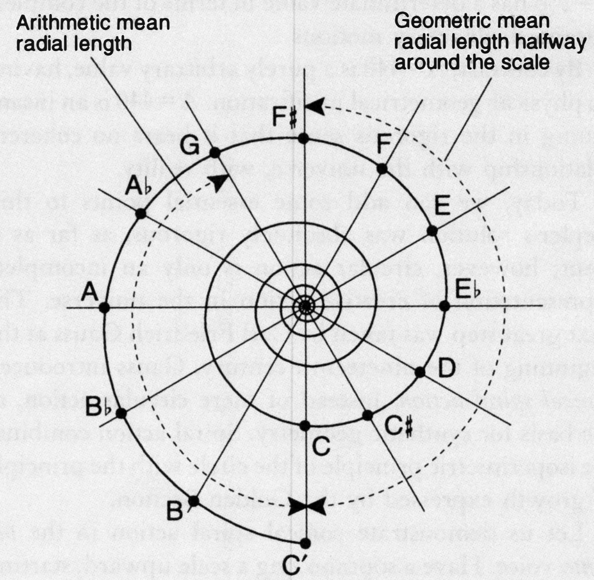


FIGURE 9. *The self-similar conical spiral projected onto a plane, showing the intervals of the equal-tempered scale.*

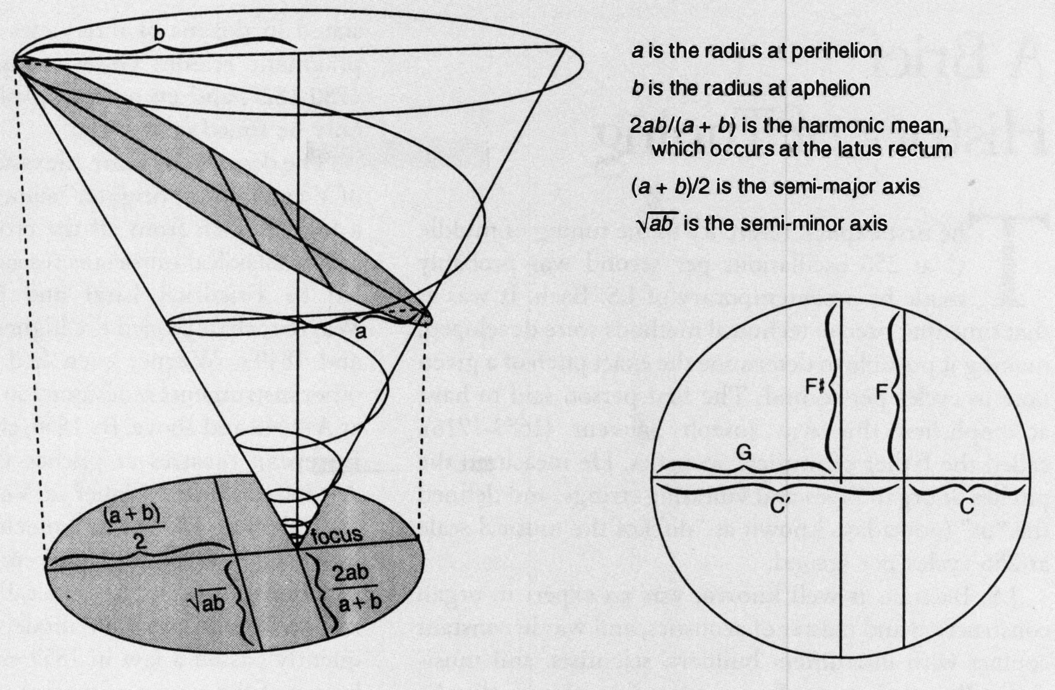


FIGURE 10. *Projection onto a plane of the ellipse formed by slicing diagonally between the circular cuts representing C=256 and C=512, showing the important division points of the octave.*

by the circles at C=256 and C=512, by slicing the cone diagonally across those two circles. The result is an ellipse. Project this ellipse onto the plane. The principal parameters of the resulting plane ellipse define exactly the frequency ratios for the most important division-points of the octave (see Figure 10):

C=256 corresponds to the perihelion of the ellipse
C=512 corresponds to the aphelion
F corresponds to the semi-latus rectum
F♯ corresponds to the semi-minor axis
G corresponds to the semi-major axis

At the same time, F, F♯, and G correspond to the harmonic, geometric, and arithmetic means, respectively, of the octave. These three means formed the basis of classical Greek theories of architecture, perspective, and music. The same notes F, F♯, and G mark the principal division of the basic C-major scale. This scale consists of two congruent tetrachords: C–D–E–F and G–A–B–C. The dividing-tone is F♯.

Physical Significance Of the Register Shift

Let us now return to our soprano. She makes the first register shift, from first to second register, exactly at this point of division. The first tetrachord, C–D–E–F, is sung in the first register, while G–A–B–C are sung in the second register. The register shift divides the scale exactly at the geometrical-mean or halfway point in the cycle of conical spiral action. The same process repeats in the next-higher octave, where the shift from second to third register of the soprano comes once again at F♯, the geometric mean.

The *bel canto* shift is a physical event of fundamental importance, and not merely a technical question for the voice. In physical terms, the register shift constitutes a singularity, a nonlinear phase change comparable to the transformation from ice to water or water to steam. An even better comparison is to the biological process of cell division (mitosis). In every case, we see that in C=256 tuning, the region of this singularity coincides with the principal *geometrical division* of conical spiral action. (Here we take the soprano voice, for musical and developmental reasons, as the fundamental reference for the human voice in general.)

Our solar system also makes a "register shift." It has long been noted that the inner planets (Mercury, Venus, Earth, and Mars) all share such common features as relatively small size, solid silico-metallic surface, few moons, and no rings. The outer planets (Jupiter, Saturn, Uranus, and Neptune) share a second, contrasting set of characteristics: large size, gaseous composition, many moons, and rings. The dividing-point between these two sharply contrasting "registers" is the asteroid belt, a ring-like system of tens of thousands of fragmentary bodies believed to have arisen from an exploded planet.

It is easy to verify that the solar-system register shift

A Brief History of Tuning

The first explicit reference to the tuning of middle C at 256 oscillations per second was probably made by a contemporary of J.S. Bach. It was at that time that precise technical methods were developed, making it possible to determine the exact pitch of a given note in cycles per second. The first person said to have accomplished this was Joseph Sauveur (1653-1716), called the father of musical acoustics. He measured the pitches of organ pipes and vibrating strings, and defined the "ut" (nowadays known as "do") of the musical scale at 256 cycles per second.

J.S. Bach, as is well known, was an expert in organ construction and master of acoustics, and was in constant contact with instrument builders, scientists, and musicians all over Europe. So we can safely assume that he was familiar with Sauveur's work. In Beethoven's time, the leading acoustician was Ernst Chladni (1756-1827), whose textbook on the theory of music explicitly defined C=256 as the scientific tuning. Up through the middle of the present century, C=256 was widely recognized as the standard "scientific" or "physical" pitch.

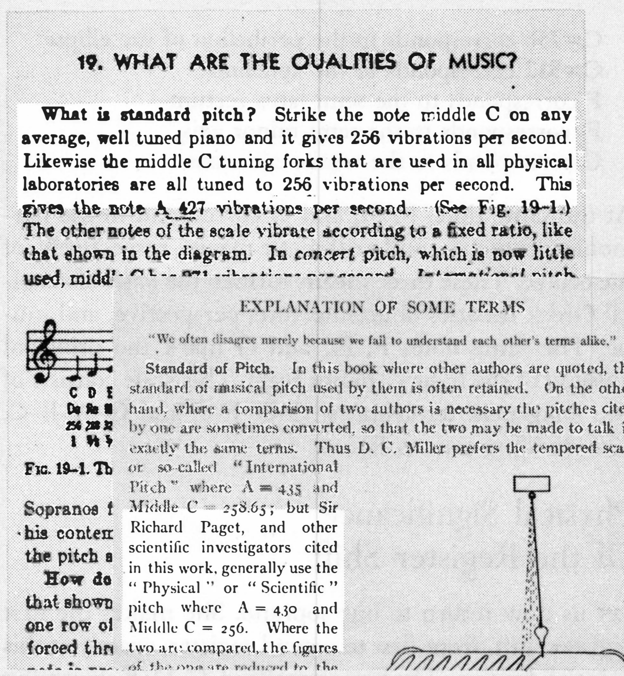
In fact, A=440 has never been the international standard pitch, and the first international conference to impose A=440, which failed, was organized by Nazi Propaganda Minister Joseph Goebbels in 1939.

Throughout the seventeeth, eighteenth, and nineteenth centuries, and in fact into the 1940's, all standard U.S. and European textbooks on physics, sound, and music took as a given the "physical pitch" or "scientific pitch" of C=256, including Helmholtz's own texts themselves. Figure 13 shows pages from two standard modern American textbooks, a 1931 standard phonetics text, and the official 1944 physics manual of the U.S. War Department, which begin with the standard definition of musical pitch as C=256.[1]

Regarding composers, all "early music" scholars agree that Mozart tuned at precisely at C=256, as his A was in the range of A=427-430. Christopher Hogwood, Roger Norrington, and dozens of other directors of original-instrument orchestras, established the practice during the 1980's of recording all Mozart works at precisely A=430, as well as most of Beethoven's symphonies and piano concertos. Hogwood, Norrington, and others have stated in dozens of interviews and record jackets, the pragmatic reason: German instruments of the period 1780-1827, and even replicas of those instruments, can only be tuned at A=430.

The demand by Czar Alexander, at the 1815 Congress of Vienna, for a "brighter" sound, began the demand for a higher pitch from all the crowned heads of Europe. While Classical musicians resisted, the Romantic school, led by Friedrich Liszt and his son-in-law Richard Wagner, championed the higher pitch during the 1830's and 1840's. Wagner even had the bassoon and many other instruments redesigned so as to be able to play only at A=440 and above. By 1850, chaos reigned, with major European theatres at pitches varying from A=420 to A=460, and even higher at Venice.

In the late 1850's, the French government, under the influence of a committee of composers led by *bel canto* proponent Giacomo Rossini, called for the first standardization of the pitch in modern times. France consequently passed a law in 1859 establishing A at 435, the lowest of the ranges of pitches (from A=434 to A=456) then in common use in France, and the highest possible pitch at which the soprano register shifts may be maintained close to their disposition at C=256. It was this French A to which Verdi later referred, in objecting to higher tunings then prevalent in Italy, under which circumstance "we call A in Rome, what is B♭ in Paris."


19. WHAT ARE THE QUALITIES OF MUSIC?

What is standard pitch? Strike the note middle C on any average, well tuned piano and it gives 256 vibrations per second. Likewise the middle C tuning forks that are used in all physical laboratories are all tuned to 256 vibrations per second. This gives the note A 427 vibrations per second. (See Fig. 19-1.) The other notes of the scale vibrate according to a fixed ratio, like that shown in the diagram. In *concert* pitch, which is now little used, midd

EXPLANATION OF SOME TERMS

"We often disagree merely because we fail to understand each other's terms alike."

Standard of Pitch. In this book where other authors are quoted, the standard of musical pitch used by them is often retained. On the other hand, where a comparison of two authors is necessary the pitches cited by one are sometimes converted, so that the two may be made to talk in exactly the same terms. Thus D. C. Miller prefers the tempered scale or so-called "International Pitch" where A = 435 and Middle C = 258.65; but Sir Richard Paget, and other scientific investigators cited in this work, generally use the "Physical" or "Scientific" pitch where A = 430 and Middle C = 256. Where the two are compared, the figures

FIGURE 13. *Into the 1940's, textbooks assumed C=256.*

Following Verdi's 1884 efforts to insitutitionalize A=432 in Italy, a British-dominated conference in Vienna in 1885 ruled that no such pitch could be standardized. The French, the New York Metropolitan Opera, and many theatres in Europe and the U.S., continued to maintain their A at 432-435, until World War II.

The first effort to institutionalize A=440 in fact was a conference organized in 1939 by Joseph Goebbels, who had standardized A=440 as the official German pitch. Professor Robert Dussaut of the National Conservatory of Paris told the French press that, "By September 1938, the Acoustic Committee of Radio Berlin requested the British Standard Association to organize a congress in London to adopt internationally the German Radio tuning of 440 periods. This congress did in fact occur in London, a very short time before the war, in May-June 1939. No French composer was invited. The decision to raise the pitch was thus taken without consulting French musicians, and against their will." The Anglo-Nazi agreement, given the outbreak of war, did not last, so that A=440 still did not stick as a standard pitch.

A second congress in London of the International Standardizing Organization met in October 1953, to again attempt to impose A=440 internationally. This conference passed such a resolution; again no Continental musicians who opposed the rise in pitch were invited, and the resolution was widely ignored. Professor Dussaut of the Paris Conservatory wrote that British instrument makers catering to the U.S. jazz trade, which played at A=440 and above, had demanded the higher pitch, "and it is shocking to me that our orchestra members and singers should thus be dependent upon jazz players." A referendum by Professor Dussaut of 23,000 French musicians voted overwhelmingly for A=432.

As recently as 1971, the European Community passed a recommendation calling for the still non-existent international pitch standard. The action was reported in "The Pitch Game," TIME magazine, Aug. 9, 1971. The article states that A=440, "this supposedly international standard is widely ignored." Lower tuning is common, including in Moscow, TIME reported, "where orchestras revel in a plushy, warm tone achieved by a larynx-relaxing A=435 cycles," and at a performance in London "a few years ago," British church organs were still tuned a half-tone lower, about A=425, than the visiting Vienna Philharmonic, at A=450.

1. G. Oscar Russell, *Speech and Voice* (New York: Macmillan, 1931); Charles E. Dull, *Physics Course 2: Heat, Sound, and Light: Education Manual 402* (New York: Henry Holt, April 1944).

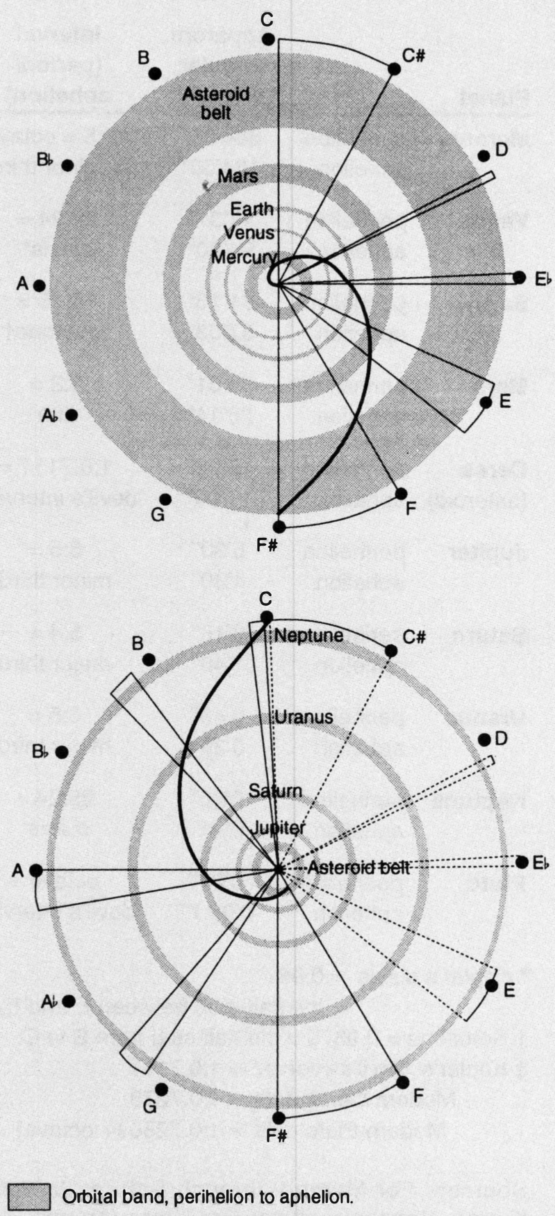


FIGURE 11. *A self-similar spiral makes one full cycle in passing from Mercury to the region defined by the overlapping orbits of Neptune and Pluto.*

falls exactly in the same, geometric-mean position, as the shift of the soprano voice in the proper C=256 tuning (see Figure 11). If we begin at the outer layer of the sun, and construct a self-similar (logarithmic) spiral making exactly one rotation in passing from that layer to the orbit of the innermost planet, Mercury, then the continuation of that spiral will make exactly one full cycle

Table I. Kepler's Harmonies of the Planets

Planet		Apparent angular velocity	Interval (period/ aphelion)
Mercury	perihelion	384′00″	12:5 = octave
	aphelion	164′00″	+ minor third
Venus	perihelion	97′37″	25:24 =
	aphelion	94′50″	diesis*
Earth	perihelion	61′18″	16:15 =
	aphelion	57′03″	semitone†
Mars	perihelion	38′01″	3:2 =
	aphelion	26′14″	fifth
Ceres	perihelion	15′06″	1:0.7111 =
(asteroid)	aphelion	11′00″	"devil's interval"‡
Jupiter	perihelion	5′30″	6:5 =
	aphelion	4′30″	minor third
Saturn	perihelion	2′15″	5:4 =
	aphelion	1′46″	major third
Uranus	perihelion	0′46″	6:5 =
	aphelion	0′39″	minor third
Neptune	perihelion	0′22″	25:24 =
	aphelion	0′21″	diesis*
Pluto	perihelion	0′24″	octave +
	aphelion	0′08.7″	"devil's interval"‡

* Kepler's diesis = 0.96
= the half-step between E and E♭.
† Semitone = 0.9375 = the half-step from B to C.
‡ Kepler's "devil's interval" = 1:0.7111
Modern Ceres data = 1:0.7278
Modern Pluto data = 1:0.7250 (+ octave)

Sources: For Mercury through Saturn: Johannes Kepler, *Harmonici mundi.* For Ceres through Pluto: modern astronomical data.

in passing from Mercury to the region defined by the overlapping orbits of Neptune and Pluto. The halfway or geometric-mean point comes exactly at the outer boundary of the asteroid belt. More precisely, if we compare the planetary spiral with our simple spiral derivation of the equal-tempered system, letting the interval from Mercury to Neptune-Pluto correspond to the octave C–C, then the planetary orbits correspond exactly in angular displacements to the principal steps of the scale. The asteroid belt occupies exactly the angular position corresponding to the interval between F and F♯; this region is where the soprano makes the register shift, in C=256 tuning. Thus, complete coherence obtains, with this tuning, between the human voice, the solar system, the musical system, and the synthetic geometry of conical spiral action.[2]

Figure 12 illustrates what happens if the tuning is arbitrarily raised, from C=256 (corresponding to A between 427Hz and 432Hz) to, for example, A=449. The soprano register shifts (at approximately 350Hz and 700Hz) lie, in the higher tuning, between E and F, rather than between F and F♯. This divides the octave in the wrong place, destroys the geometry of the musical system, destroys the agreement between music and the laws of the universe, and finally, destroys the human voice itself.

If we arbitrarily changed the "tuning" of the solar system in a similar way, it would explode and disintegrate! God does not make mistakes: Our solar system functions very well with its proper tuning, which is uniquely coherent with C=256. This, therefore, is the only scientific tuning.

NOTES

1. Bernhard Riemann, "Über die Fortpflanzung flacher Luftschwingungen von endlicher Weite," in *Gesammelte mathematische Werke,* ed. H. Weber (Leipzig, 1876), pp. 145-164. English translation: "On the Propagation of Plane Air Waves of Finite Amplitude," *International Journal of Fusion Energy* (1980), Vol. 2, No. 3.
2. Recent work by the late Dr. Robert Moon and associates has extended this coherence to the "microcosm" of subatomic physics.

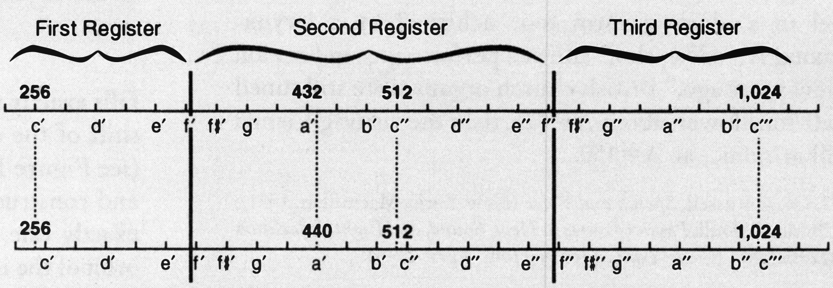


FIGURE 12. *At A=432 or below (top scale), the register shift occurs between F and F♯; at A=440 or above (bottom scale), it is forced downward to between E and F.*