

# Asteroid Harmonics

*Research Update*

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## ***Abstract***

Recent work on asteroid harmonics has turned up some very interesting results. In particular, it has revealed that there is a structure to the *centers of mass* of the asteroid orbits. These centers of mass themselves form nearly perfectly circular rings, with centers that mostly lie in the neighborhood of a triangle formed by the centers of mass of the sun, Mars, and Jupiter.

## ***Prefatory glossary***

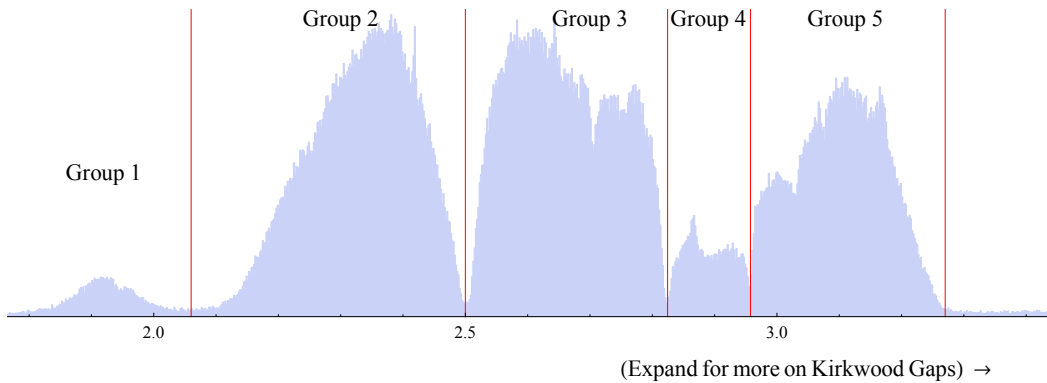
There are some basic terms used in describing planetary orbits that are needed to discuss the work on asteroids. The characteristics that determine the orbit of an asteroid (or planet) are known as the *orbital elements*. These elements include:

- $a$  – semi-major axis. This is the average distance of the body from the sun, and half the long diameter of the ellipse.
- $e$  – eccentricity. This is the distance between the sun and the center of the planet's orbit, expressed as a portion of the semi-major axis.
- $i$  – inclination. The planets and asteroids do not move in a common plane. The inclination of an asteroid is how many degrees its orbital plane is tilted with respect to the ecliptic (the plane of the earth's motion around the sun).
- $\Omega$  – longitude of the ascending node. Any orbital plane intersects the ecliptic in two places, known as nodes. The ascending node is the one where the body moves from being on the south side of the ecliptic, to the north side. Its longitude is the location, measured in degrees with respect to what's sometimes called  $0^\circ$  Aries, the spot on the ecliptic occupied by the sun on the spring equinox.
- $\omega$  – argument of perihelion. This is the number of degrees, measured from the ascending node, of the perihelion, the spot which is the closest the heavenly body comes to the sun.
- $\Omega + \omega$  – longitude of perihelion.

## ***Dividing the asteroids***

In an attempt to apply Johannes Kepler's harmonic approach to the asteroids, the most immediate problem to present itself is that while Kepler had a small number of orbits to work with (six), there are so many asteroids already discovered (and so many more that are as-yet unobserved), that they create an almost continuous distribution of asteroids and asteroid orbits. Since a continuous distribution doesn't lend itself to analysis by the methods used by Kepler, it was necessary to look for ways of dividing up the asteroids into potential functional wholes which could then be treated as single components of the system of asteroids as a totality. While it may not be possible to find one orbit that characterizes the asteroids as a whole, perhaps a number of characteristic orbits could be found.

The first division which suggested itself is the "Kirkwood gaps." These are values of  $a$ , the semi-major axis, at which no (or very few) asteroids have been discovered. The distances correspond to periodic times that are in harmonic resonance with that of Jupiter. The six red lines below, correspond to orbits whose periodic times would be  $\frac{1}{4}$ ,  $\frac{1}{3}$ ,  $\frac{2}{5}$ ,  $\frac{3}{7}$ , and  $\frac{1}{2}$  that of Jupiter. In between these gaps, five main groups suggest themselves. These will be referred to as "Kirkwood groups." This image indicates the density of asteroids having different semi-major axes (indicated on the lower axis):



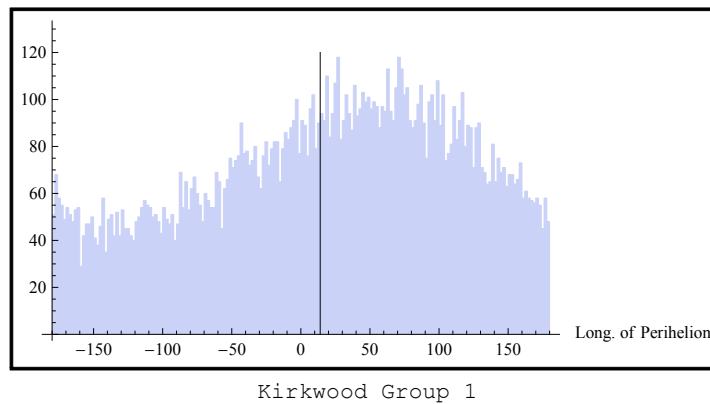
While these gaps exist when looking at the semi-major axis, they do not represent gaps in space: distances from the sun where no asteroids are found. Such gaps do not exist. See this video showing an assortment of asteroids from the five groups in order, noticing the overlap. Some of the asteroids in group 4 come closer to the sun than do most in group 1! (Video: <http://www.youtube.com/watch?v=5U4VuKL2LUE>)

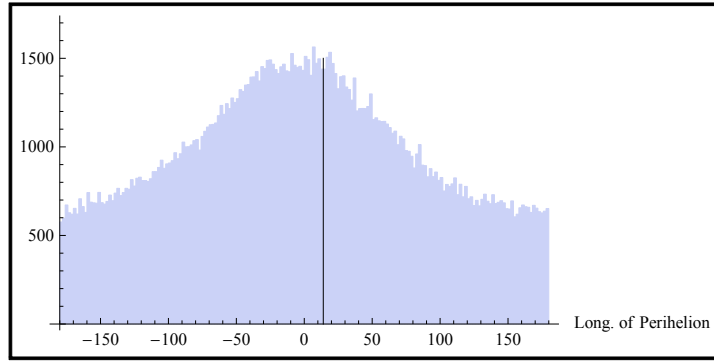
### ***Characterizing the groups***

The attempt to develop singular characteristic (or average) orbits appears to work much better when groups are considered one at a time, rather than looking at all the asteroids at once. For example, in each “Kirkwood group,” we found that the average aphelion and perihelion actually corresponded reasonably well to the average eccentricity, something that is *not* true for the asteroids as a whole.

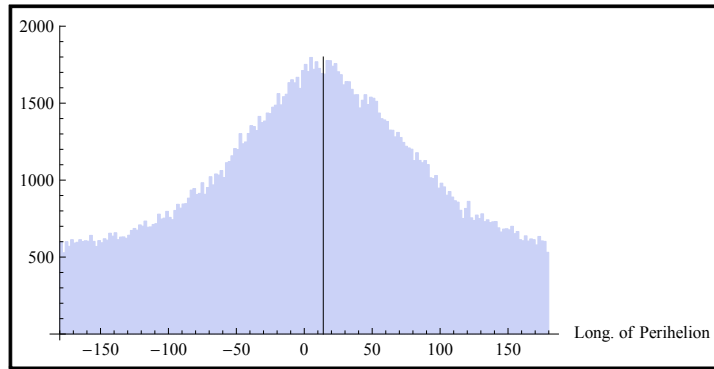
### ***Broadening the investigation***

To apply Kepler’s specific harmonic method, the only parameters considered for an orbit are the semi-major axis ( $a$ ) and the eccentricity ( $e$ ), since they alone determine the relative speeds of heavenly bodies at their closest and farthest distances from the sun. But to broaden the investigation, all parameters were considered. The histograms for the longitude of perihelion are very revealing:

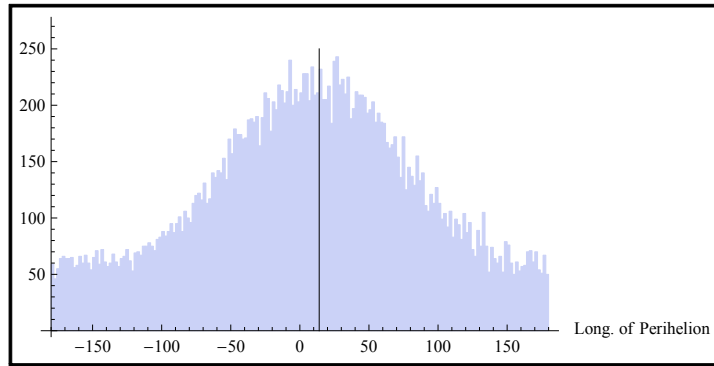




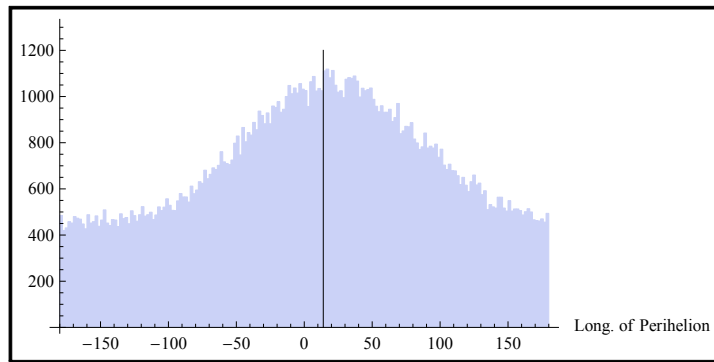
Kirkwood Group 2



Kirkwood Group 3



Kirkwood Group 4

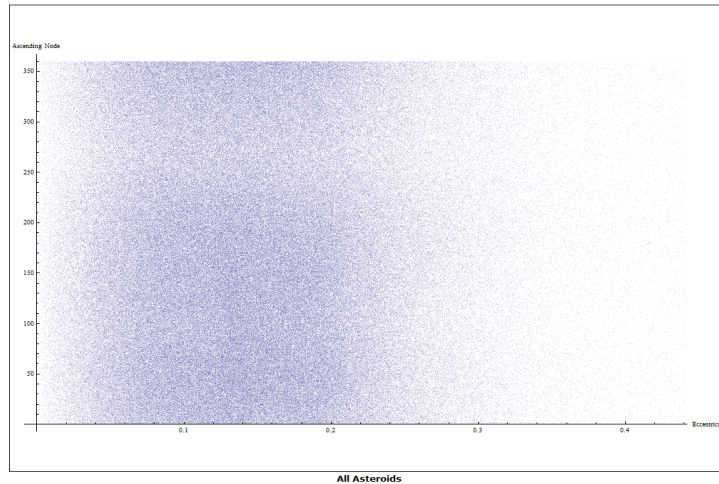


Kirkwood Group 5

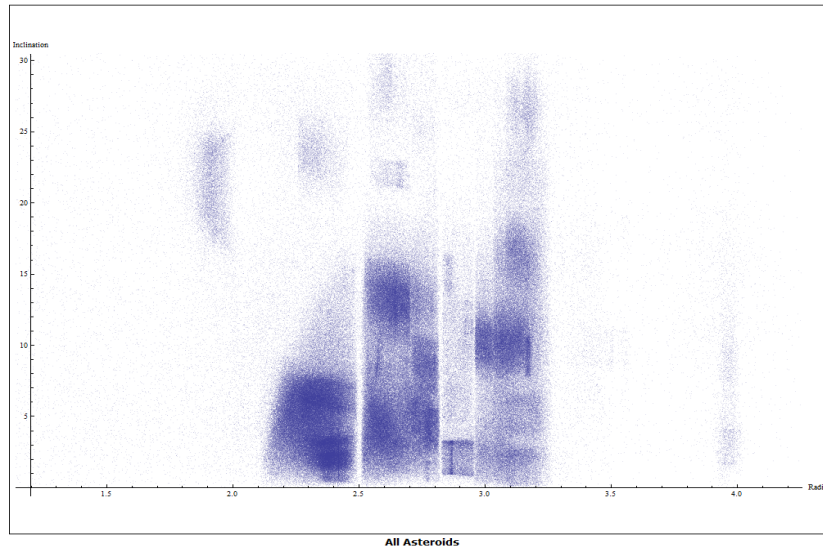
These histograms show that all of the five groups have their perihelial longitudes roughly aligned. With what might they be aligned? The black line drawn in on the histograms represents the longitude of perihelion of Jupiter. Therefore, the asteroids are being pulled on by (or, at least, have *something* in common with) Jupiter (or something in Jupiter's direction). This is not hugely surprising, considering how large Jupiter is and how (relatively) close it is to the asteroids, but the tendency towards Jupiter is certainly *not* obvious from videos of asteroid motion.

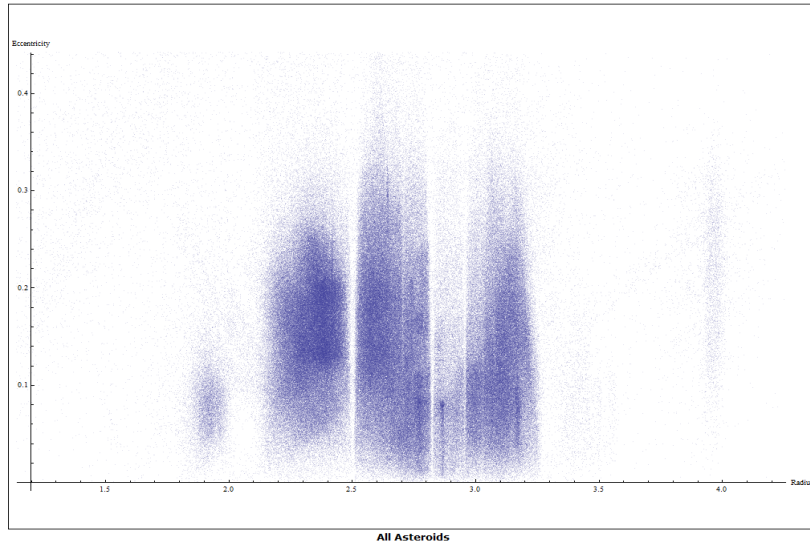
### More parameters

At this point, I decided to make charts of every comparison of parameters, for all the asteroids together, and for each group individually. Some didn't indicate much, such as this one, which compares eccentricity and longitude of the ascending node. The graph is more dense on the left side, indicating that most asteroids have eccentricities less than 0.2, but no particular structure appears in the comparison of these two parameters.

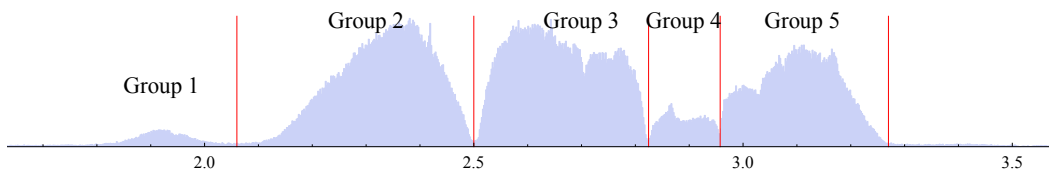


By contrast, consider these scatter-plots of semi-major axis (labeled "radius" here) with inclination and eccentricity. Every dot you see represents an asteroid orbit. Darker regions mean there are more asteroids with that combination of inclination (or eccentricity) and semi-major axis:





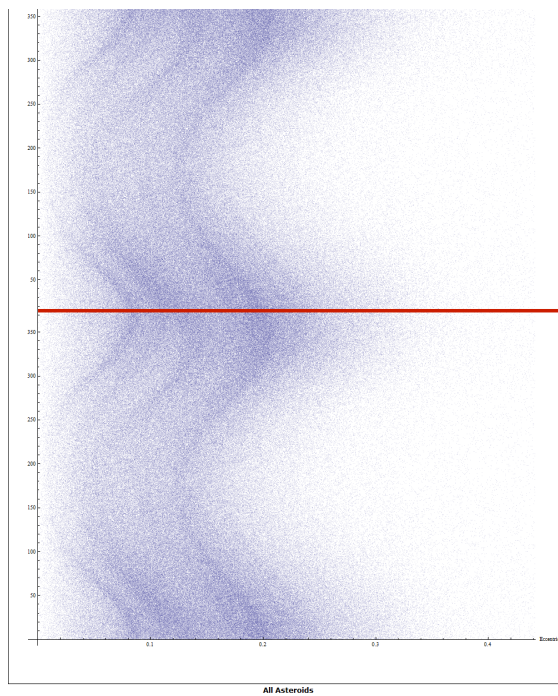
They are similar to the very first histogram of semi-major axis alone:



The densities of the histogram are much more lively in the scatter-plots. The inclination scatter-plot gives another dimension to look at for divvying up the asteroids, breaking up Group 3 into five (or more) different clumps. What will we learn by separately treating each of these segments of the space of asteroids?

***The big clue***

The eccentricity–longitude of perihelion scatter plot sheds new light on the longitude of perihelion histograms considered above:

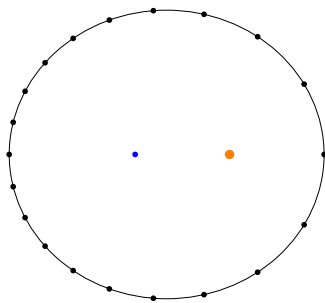


This chart, which has  $720^\circ$  along the y-axis (it repeats to make the sinusoidal shape clearer), not only shows a density of asteroids around the perihelion of Jupiter (indicated by the red line at  $375^\circ$ ), but shows definite bands of coherent asteroid clumps whose eccentricity varies with perihelion longitude. The eccentricity is greatest for those asteroids sharing a perihelion with Jupiter, and smallest for those asteroids of opposite orientation.

### The Center of Mass

To make the next step, we have to look at entire orbits being affected by Jupiter. To do this, we'll use one location to characterize each orbit. Rather than the geometrical center of the orbits, we'll use a time-weighted center of mass.

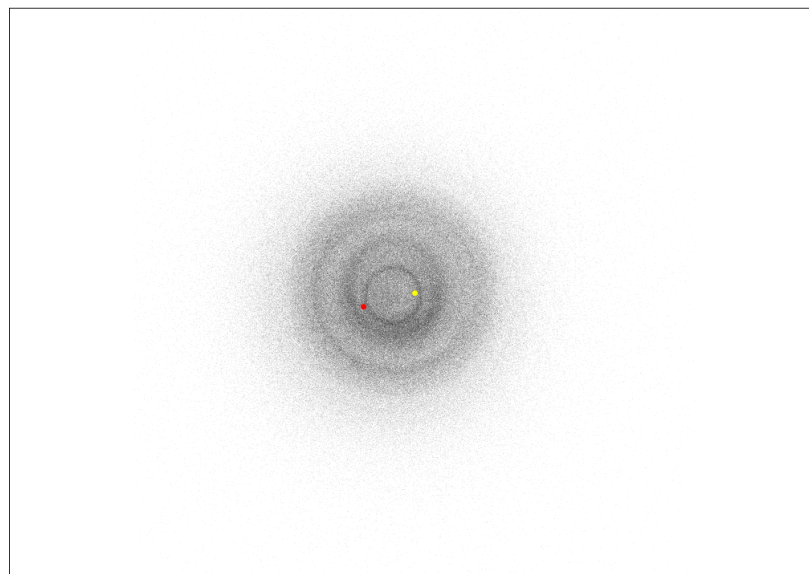
When Carl Gauss looked into the interactions between different heavenly bodies, he realized that as long as their periods are not in a resonant ratio, it was possible to smear out the mass of a planet into an elliptical ring, where the density was based on how quickly the planet moved, and use the unchanging ellipse instead of the changing position of the planet, to look at interactions between different orbiting bodies. For our purposes here, we can take an asteroid orbit, break it into 20 pieces (by time), and then take the middle of those 20 asteroid locations.



Ellipse with  $e=0.4$  with the sun indicated by the orange spot. The twenty marked spots are spaced evenly by the time for the body to move between them on its orbit. The average location of those points, their center, is marked by the blue spot. This is not the same location as the geometrical center of the ellipse.

This is one way of representing an entire orbit by one point.

This created a "center of mass" of each of the asteroids, which seemed more real than their purely geometric centers. Planets move in ellipses, with the sun at one of the foci. The center of mass lies almost exactly between the center of the orbit and the other focus, varying slightly by the eccentricity of the orbit. Next, I started to look at these centers of mass for the different "Kirkwood groups", expecting a vague prevalence in the direction of Jupiter. But what appeared was nothing short of astounding! Here are the centers of mass for all of the asteroids, in space, with the sun in yellow and the center of mass of Jupiter in red:



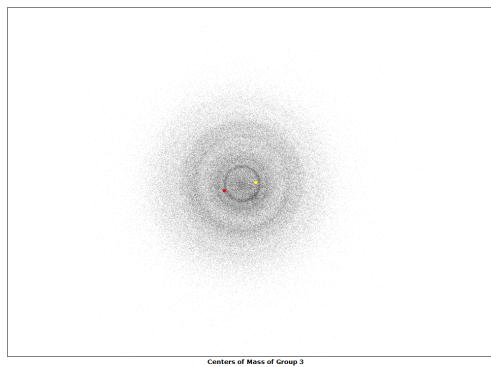
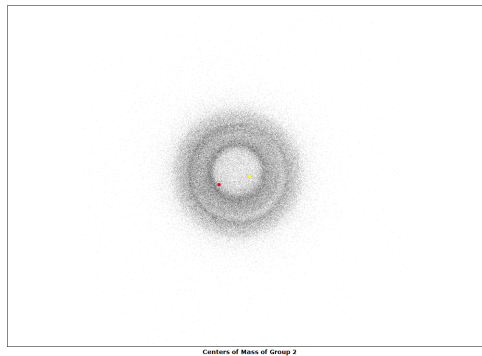
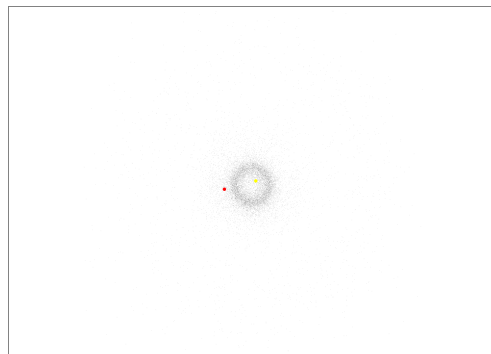
Centers of Mass of All Asteroids

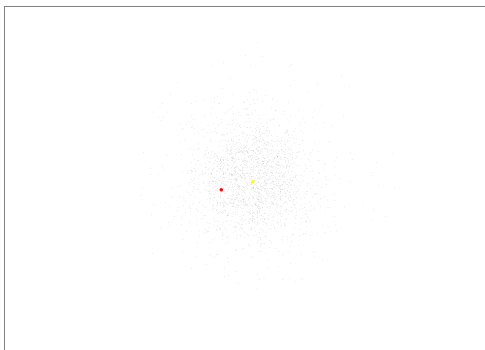
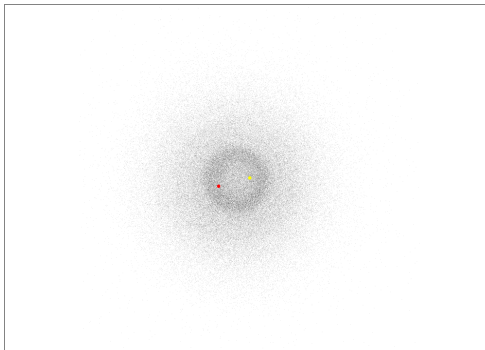
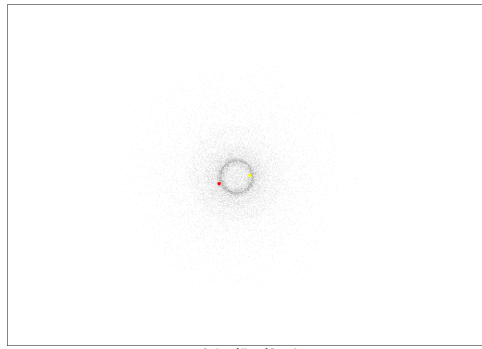
As you can see, the centers of mass are themselves forming circles! What could be causing this? Remember, this is not an image of rocks orbiting around something. This imagery is looking at points associated with orbits as wholes, not as the individual asteroids that appear somewhere on those orbits. Rather, these are practically motionless points distributed in rough circle.

True, these centers of mass are moving (since all orbits change over time), but they are certainly not moving quickly at all! Can these circles be thought of as static rings? Or are the centers moving faster than is typically imagined? *To answer this, we'll have to determine how the orbits of the asteroids change over time!* But how can this be done? One current possibility, that requires only new calculations but no new observations, would be to use the historical record of observations to recalculate the asteroids' orbits at times in the past. That is, calculate the orbits of the known asteroids ten years ago without any of the observations of the past decade, and compare them with their orbits today. This could indicate how the centers of mass have changed over the past ten years. Although it would be surprising if these differences were large, compared to the uncertainty in measurements, having a large number of asteroids will make it possible to tease out significant changes despite the noisy data. Unfortunately, the historical record of such observations for most asteroids only reaches back about one century, while those asteroids discovered much longer ago, such as Ceres, Vesta, and Pallas are orders of magnitude larger than the average asteroid and may not behave in a way common to the others.

### ***Animating the Centers of Mass***

First, let's compare the centers of mass for the different Kirkwood Groups:





Now, we will animate the process, by moving away from the sun continuously, rather than discretely, by group. This video ([http://www.youtube.com/watch?v=6wN8xfy\\_zHo](http://www.youtube.com/watch?v=6wN8xfy_zHo)) shows the continuous progression, with the Kirkwood gap histogram in the background. Here is another version (<http://www.youtube.com/watch?v=uWXmys30Eqk>), in which the circles appearing in the distribution of centers of mass are marked in as they appear.

To view this process from a slightly different vantage-point, this animation (<http://www.youtube.com/watch?v=8GJggtTJrCo>) again selects asteroids by moving along the semi-major axis, but this time, the calculated center of mass of these centers of mass is itself indicated as the moving dark spot. Other centers of mass are included: Mars (red), Jupiter (magenta), Saturn (teal), Ceres (darkest black), Vesta (lighter black), and Pallas (lightest gray dot).

To take the animation from a different standpoint, this video (<http://www.youtube.com/watch?v=byf3lpnSHMM>) fixes the semi-major axis  $a=2.786$ , and animates by changing the inclinations of asteroids that are included in each frame. As you can see, the rings are distinguished by their inclinations. Perhaps these are the same distinctions seen in the semi-major axis – inclination scatter plot?



### *Next steps*

These circles are an excellent candidate as a criterion for splitting up the asteroids into groups, which may have interesting harmonic relationships between them. After breaking the asteroids up into these rings, all the scatter plots could be remade for those groups. Questions that arise include:

- What could possibly be causing these centers of mass to lie in a circle?
- And what are the centers of the circles?
- Does their orientation point to a location within the solar system itself? What lies in the sidereal direction of their centers? Could the crab nebula be involved in a non-gravitational way?

Other directions for follow-up include:

- Breaking up the asteroids by which center-of-mass circle they appear in, or by  $a$  and  $i$ .
- A general search for celestial phenomena in the directions suggested by these rings, to consider causes outside the Solar system.
- If size estimates could be made for the asteroids, it would allow for more accurate determinations of the centers of mass of groups of asteroids. Right now, it is based on presuming they have equal sizes.

Overall, this study definitely gives the indication that we are looking at field phenomena, not just discrete asteroids and observations of them. What suggestions might arise from taking a hydrodynamic view of this apparent field?